A block diagram approach to macroeconomic dynamics, and why IS/LM is fatally flawed

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A dynamic model of an individual, and then an aggregate (sector), economic unit is developed. This model and other building blocks are employed to create macroeconomic models represented through block diagrams. A simulation tool based on block diagram representation is applied to a simple textbook economy with firms and households. Finally, a dynamic extension of the IS/LM static model is presented in block diagram form, and it is demonstrated through the dynamic extension that IS/LM's way of treating money stock is flawed to a degree that implies that IS/LM must be discarded.

JEL classification: B50, C02, C60, C65

1. Introduction

Dynamics are mathematically and conceptually much more complicated than (comparative) statics: Algebraic equations are substituted with differential equations. These equations are difficult to work with in the sense that one cannot – as in a static framework – find graphical solutions to them without computer-implemented solution algorithms.

Furthermore, it is difficult to gain insights about the properties of a system by inspecting its differential equations. It will hopefully be demonstrated that the method of representing the system graphically through block diagrams lends itself easier to such insights. Representing a system this way may be considered an interface between the user and the differential equation based model. This paper – among other things – tries to convey the usefulness of the (graphic) block diagram approach.

The structure of the paper is as follows: We start in section 2 with choosing the simplest possible dynamic model: an economic unit with the approximate dynamics of a vessel with money flowing through it. This is in the "hydraulic Keynesian" tradition of A.W Phillips (1954,1957). Vessel dynamics is compared to an alternative of "pipeline dynamics", and argued to be superior.

Some basic control systems concepts and tools for continuous-time modeling are introduced in subsection 2.1. This subsection may be skipped or fast browsed by readers with this type of background.

We then argue in subsection 2.2 that the "vessel dynamics" model is not only useful in the sense that it is the simplest one that gives meaningful behaviour (an "Occam's razor" choice which was Phillips' reason), but that it may be additionally justified when one considers that a sector is the aggregate of a large number of individual units. A theorem about this is presented and proved.

In section 3 two basic "textbook" macro models are presented and discussed using the earlier introduced concepts and tools.

Section 4 argues for a fairly dramatic claim, a claim that may be the more controversial since the argument given is quite simple. The claim is that the IS/LM model is fundamentally inconsistent and therefore should be discarded.

2. A money stock/flow model for a generic economic unit.

An economic unit in our terms may be a household, a firm, a bank, a government. The generic economic unit concept is shown in figure 1:

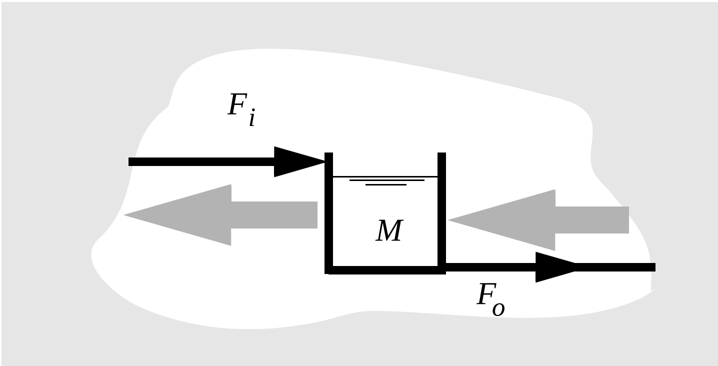

Figure 1

The unit may be compared to a “vessel” or “reservoir” with varying volume of “fluid”. Money flows F_i (in) and F_o (out) are shown as black arrows. While money in the real world moves between units in discrete “packets”, we will consider money flows to be continuous. This is reasonable for the time scale (weeks, months, years) of the dynamics that is to be considered. Real flows (labour, goods, services) are suggested by the thick shaded arrows in the figure. The grey shaded area surrounding the unit is simply the aggregate of all other units, i.e. the macroeconomic system.

Money stock M for the unit is the volume in the vessel at a given instant. Its size depends on the unit’s precautionary, speculative and transaction motives.

Money stock may also be interpreted as due to a neccessary *decision+action time delay* τ for the unit before received cash is passed on again.

For the special case with $F_o = F_i = F$ constant, M will also be constant. We may then think of the time delay in terms of a specific “particle” of money arriving at the inlet, appearing at the outlet τ time units later. We have

$$M = F\tau \text{, or } \tau = M/F \tag{1}$$

From (1) follows that a *local velocity of money* is:

$$v = 1/\tau \tag{2}$$

The delay associated with flows in general (as in process plants, pipelines, etc.), will in the case of money be the time a given amount spends between arrival and departure *at a given unit*. Flows *between* units may be reckoned as immediate. Thus money always resides at some unit.

We now introduce the unit step function $\mu_1(t)$ and the corresponding step response *k(t)*. The step function simply means that at time $t = 0$, an incoming flow of money with amplitude = 1[currency unit/time unit] begins, and the flow F_o resulting from this specific input, is the step response. For the unit we could conjecture that the money flow is delayed exactly τ time units, resulting in the trivial step response shown to the right in figure 2 (assuming that the unit starts out with zero money stock):

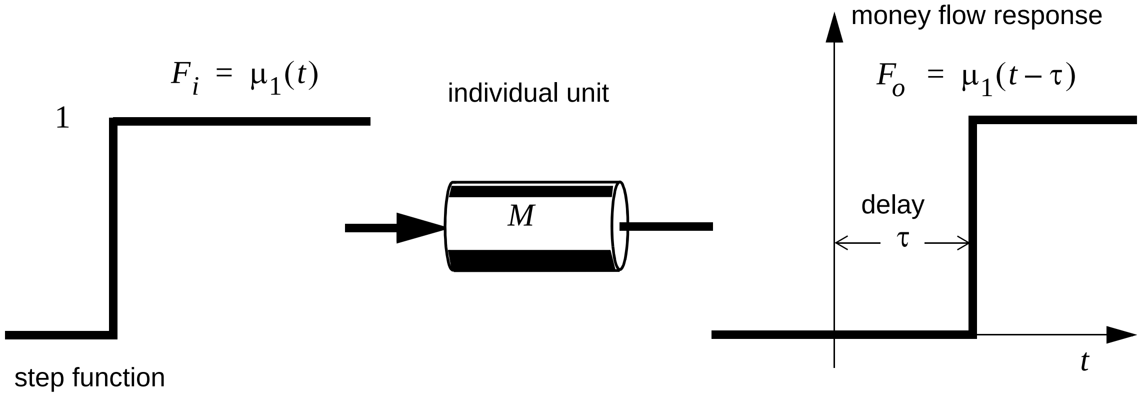

Figure 2

Such a response would have occured if the unit had been comparable to a “pipeline”, as suggested in the figure. The vessel analogy, however, is obviously more realistic, and its response is shown in figure 3. Fluid has to rise in the vessel to build up the neccessary “pressure” before an outflow starts. More specifically, we assume the following dynamics: The unit reacts to a monetary step function type flow with a time-dispersed exponential spending response asymptotically approaching the incoming flow level.

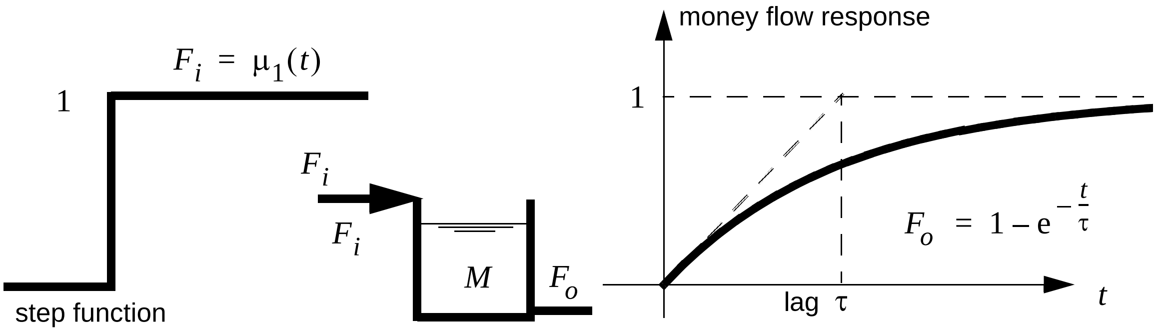

Figure 3

The term τ is now not a time delay, but what in systems theory lingo is called a time *lag*. Geometrically, it corresponds to the position along the time axis of the intersection between the tangent at $t = 0$, and the horizontal asymptote. The unit react to a sudden incoming money flow by gradually increasing its spending, and the parameter describing the speed of adjustment is τ. When spending flow F_o (theoretically) has reached the asymptotic level, we have equilibrium. The money stock of our unit (the “volume of the vessel” in figure 1) must be the integrated difference between income- and spending flows. We then have the differential equation

$$\dot{M}(t) = -F_o(t) + F_i(t) \quad (3)$$

At the same time we demand that the step response $F_o\,(t)$ shall be as in figure 3:

$$F_o(t) = 1 - e^{-\frac{t}{\tau}} \quad (4)$$

If we choose

$$F_o(t) = \frac{1}{\tau}M(t) \quad , \tag{5}$$

it may be shown that both (3) and (4) are satisfied. We now observe that as long as we confine ourselves to $F_o(t)$, (5) corresponds to definitions (1) and (2) for respectively time delay and money velocity. These definitions were based on the the very unrealistic assumptions of constant and equal in- and outflows, and a pipeline model. These assumptions may now be rescinded. The absence of $F_i(t)$ in (5) is reasonable, since the unit exercises control only over $F_o(t)$.

Equation (5) is intuitively satisfying in the sense that the outgoing flow is proportional to money stock, which can be regarded by physical analogy as a “pressure” driving this flow (pressure is proportional to fluid level in a vessel, which again is proportional to fluid volume when the vessel is cylindrical). The larger the time lag τ, the less flow F_o for a given M, i.e. a large time lag (lower velocity) means that money has to accumulate significantly at the unit before the unit increases spending. The parameter τ is the first behavioral assumption for our generic unit. One may let τ be influenced by other system variables, for instance let it increase sharply in a recession/depression (increased liquidity preference) or decrease with increasing interest rates. Such modifications will make a model consisting of such units nonlinear. But in this paper we confine ourselves to the simple assumption of constant τ.

2.1 *The Laplace transformation and block diagrams*

(This subsection may be skipped or browsed by readers familiar with control systems literature and concepts.)

Finding time responses for systems of the type introduced above, requires solution of linear differential equations. A tool that makes this task easier, both in the stage where the problem is to construct and understand a model, and in the subsequent solution (or numerical simulation) stage, is the *block diagram*. We will later employ this tool extensively. Block diagrams are based on the Laplace transformation, which is described in most undergraduate mathematical textbooks. The main advantage of the Laplace transformation is that differential equations are substituted with algebraic equations. We will develop the topic through the example given by eqs. (3)-(5). Substituting (5) in (3), we get:

$$\dot{M}(t) = -\frac{1}{\tau}M(t) + F_i(t) \tag{6}$$

Laplace transforming both sides leads to

$$sM(s) - M_0 = -\frac{1}{\tau}M(s) + F_i(s) \tag{7}$$

where s is the Laplace transformation[1] free variable. M_0 is the initial value for $t = 0$.

This equation may be solved for $M(s)$,

$$M(s) = \frac{\tau}{1 + \tau s}F_i(s) + \frac{\tau}{1 + \tau s}M_0 = h_{mi}(s)F_i(s) + \frac{\tau}{1 + \tau s}M_0 \quad (8)$$

Here $h_{mi}(s) = \frac{\tau}{1 + \tau s}$ (9)

is the *transfer function* from $F_i(s)$ to $M(s)$, enabling us to find the response $M(t)$ when $F_i(t)$ (i.e. also $F_i(s)$) is given.

Consider the case in figure 3; which has initial zero money stock, $M_0 = 0$, and a step input flow $\mu_1(t)$, which has the Laplace transform $1/s$. Then (8) gives

$$M(s) = h_{mi}(s)\frac{1}{s} = \frac{\tau}{(1 + \tau s)s} \quad (10)$$

which has the inverse transform

$$M(t) = \tau\left(1 - e^{-\frac{t}{\tau}}\right) \quad (11)$$

This is the step function response for the money stock. Employing (5), we get the spending flow step response $F_o(t)$ already given in (4):

$$F_o(t) = 1 - e^{-\frac{t}{\tau}} \quad (12)$$

The transfer function from $F_i(s)$ to $F_o(s)$ also follows from (5), giving

$$h_{oi}(s) = h_{mi}(s)\frac{1}{\tau} = \frac{1}{1 + \tau s} \quad (13)$$

So far on the dynamics of a unit with initial zero money stock and a constant inflow of money starting at $t = 0$. If we alternatively consider an unit with a certain initial money stock M_0 but no income, i.e. $F_i(t) = 0$, then we may also find the time path of $F_o(t)$. From (8) we now get

$$M(s) = \frac{\tau}{1 + \tau s}M_0 \quad (14)$$

which inverse transformed is $M(t) = M_0 e^{-\frac{t}{\tau}}$, leading to

$$F_o(t) = \frac{1}{\tau}M(t) = \frac{M_0}{\tau}e^{-\frac{t}{\tau}} \quad (15)$$

We conclude that our unit spends its money following a decaying exponential curve, which seems reasonable in a situation with zero income. See figure 4:

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1. Both functions of time and Laplace transforms are written with the same symbol. The context, or explicitly written dependence on *t* or *s* will suffice to distinguish between them. Note also that *s* is here not the savings coefficient. We avoid confusion in this paper by using the propensity to consume instead, $c = 1 - s$.

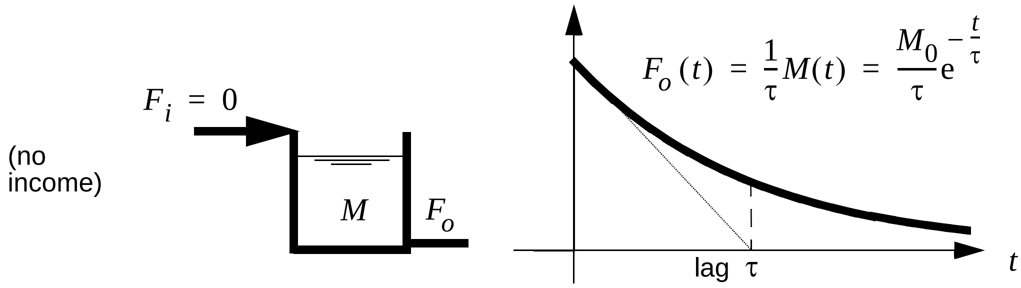

Figure 4

We may now introduce the block diagram, which embodies the same information as that represented through differential equations, but which is better for understanding a system. This is the rationale for modeling and simulation packages such as Simulink (Mathworks, 2007) being based on block diagram description.

Consider figure 5:

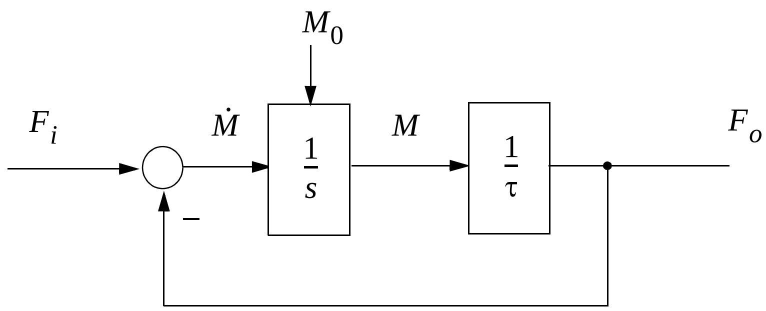

Figure 5

The rules for drawing and interpreting this diagram are as follows:

1. The variable exiting a rectangular block is the product of the variable entering the block and the expression within the block. Thus we have (5): $F_o = M/\tau$.
2. The small dot to the right signifies a *branching point*. This means that the variable F_o is both an output of the system, and also fed back to the system's input side. In this case we have a negative feedback.
3. The circle to the left is a *summation point*. The arrow leaving the circle is the sum of arrows entering the circle. An arrowhead with a minus sign associated with it, means that the variable corresponding to this arrow is to be subtracted in the summation. Thus we have (4); $\dot{M} = -F_o + F_i$.
4. A block of the type $1/s$ signifies (in accordance with rule 1) that the variable exiting this block is $1/s$ times the variable entering it. But dividing by *s* in Laplace symbolism signifies integration: $M = \int \dot{M} dt$. The block of the type $1/s$ is accordingly called an *integrator*.
5. The vertical arrow on top of the integrator specifies the initial value of the output variable from the integrator. This arrow is often rescinded for convenience, or if the corresponding initial value is zero.

Note that we have avoided signifying whether F_i, M, F_o in the block diagram depend on the Laplace variable *s*, or time *t*. The reason for this is that the block diagram may represent *both* the Laplace-transformed case and the time domain case. We just have to keep in mind that the integrator in the Laplace-transformed interpretation of the block diagram means multiplying with $1/s$, as opposed to when we want the diagram to represent relations between time-varying variables.Then the block $1/s$ is an *integration operator* - it stenographically signifies the relationship $M = \int \dot{M} dt$. For more on operator interpretation of *s*, see for instance Rowell (1997, 207 - 211).

The block diagram in figure 5 corresponds exactly to the model given by equations (3) and (5). By inspecting the figure, we see how the spending flow F_o is caused by "pressure" from money stock M, while F_o at the same time feeds back negatively and depletes the same money stock.

The block diagram in figure (5) is called an *elementary* block diagram, because it contains only "simple elements" like integrators (one in this case) and constants ($1/\tau$ in this case). Such a block diagram may be changed (*reduced*) into an equivalent (in an input/output sense) diagram, where a simpler structure is achieved at the cost of more complex expressions in the blocks that remain after the procedure. For our example, the reduced block diagram turns out to be as shown in figure 6. This diagram, reduced to only one block, simply results in the transfer function $h_{oi}(s)$, see (13).

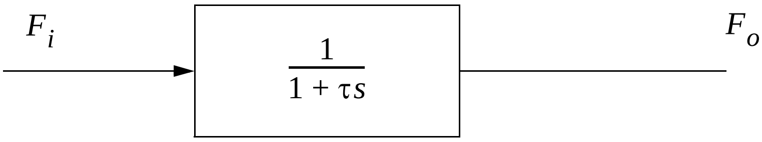

Figure 6

Manipulating block diagrams into different but equivalent diagrams will be done later in examples.

2.2 An aggregate unit

We now will consider a generic aggregate unit (a sector), which consists of a large number of individual units. Such an aggregate unit may for instance represent all households, or consumption goods firms, or all firms, or all banks, etc.

Let us confine the discussion to units within a given sector. Individual units there will of course have different "sizes" in the sense that money stock and flow magnitudes will vary widely. But we assume that (5) holds for all units in a given aggregate, i.e. that the money stock of a specific unit is proportional to the spending flow from the unit, by a common factor τ. Thus all units in a given sector may be represented by the transfer function (13).

We furthermore assume that any (in an average sense) individual unit's outgoing money flow is divided into fractions ρ (out of the sector) and $(1 - \rho)$ (to other units within the sector), where $0 < \rho < 1$. This is illustrated in figure 7:

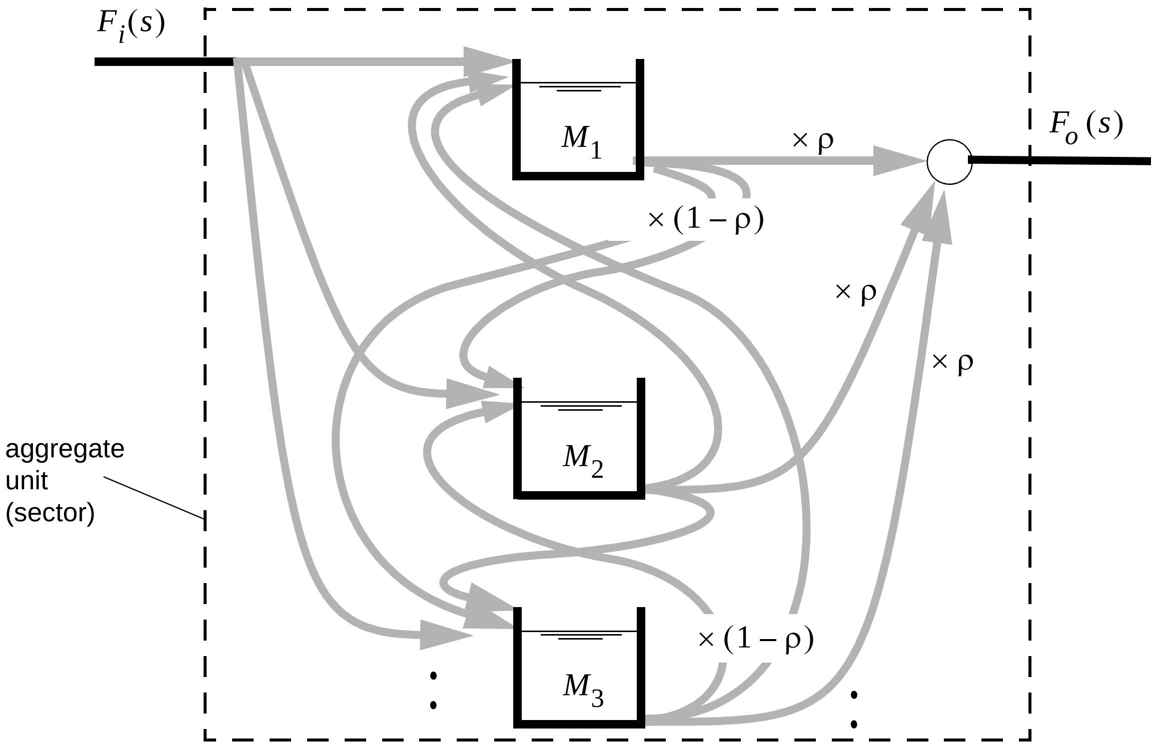

Figure 7

Note that this is a “physical” flow chart, not to be confused with the mathematical block diagram introduced earlier. The shaded arrows indicate a network of interactions, where any individual unit in principle interacts with any other unit. Our interest is still focused on two aspects, input-output characteristics of the aggregate unit, and the dynamics of aggregate money stock. The surprisingly simple result is that - under the above assumptions - the transfer function for the aggregate unit turns out to be

$$h_a(s) = \frac{1}{1 + T_a s}, \text{ where } T_a = \frac{\tau}{\rho} \geq \tau \qquad (16)$$

Before proceeding with the proof, some comments to indicate that this result is intuitively satisfying. Let us first consider a type of sector where the population of units have a low volume of monetary transactions between them, even if the number of units may be large: A case in point is the aggregate of all households. In this case ρ is close to unity. Referring to figure 7, this means that the units are simply laid out “in parallell”, with negligible flows between them. Money arriving at a specific unit will emerge from the the unit and also the aggregate, without having to “percolate” via other household units first. Thus one should expect the aggregate to have the same fast response as an individual unit. This also fits with (16), since $T_a = \tau$ in the limit when $\rho = 1$.

For the firm sector, we will have $\rho < 1$, since each firm will direct a significant part of its money outflow to other firms, not out of the sector.

The extreme case $\rho \ll 1$ is when the “aggregate unit” is such that units mostly do their transactions with other units *within* the aggregate. This case fits well with what financial sectors have developed into for the last decades. An outside unit who injected money into such an aggregate, would – if she had the means to “trace” that packet of money – observe that it would take a very long time before the last residue of the injected amount emerged from the aggregate. It is consistent with (16), where a small ρ means a large lag T_a, giving just the type of low-amplitude, drawn-out response that seems reasonable.

We will now prove (16).

Proof: In deriving the transfer function for the aggregate unit, we may assume that the outside incoming monetary flow arrives at one unit only, because of the symmetry between the units, and because of the superposition principle that applies to a linear system: If the incoming flow was instead distributed between several units, the resulting response would be the sum of responses to each component of the incoming flow, transmitted through identical transfer functions, which would then sum up to the result we get when the incoming flow is considered to arrive at a single unit only.

Consider the structure in figure 8. It is a block diagram with transfer functions. This block diagram accounts for the way an incoming monetary flow branches through the aggregate of units. As already argued we may assume that the flow enters at one single unit, the uppermost in figure 8. This results in a spending flow which, according to figure 7, is partioned into a share ρ leaving the aggregate, and a share $1-\rho$ to another unit within the aggregate. This share again results in a flow that is partioned into a share ρ leaving the aggregate, and a share $1-\rho$ to another unit within the aggregate, and so on. The transfer function

$$h_a(s) = F_o(s)/F_i(s) \tag{17}$$

is indicated in the figure by the light shaded area.

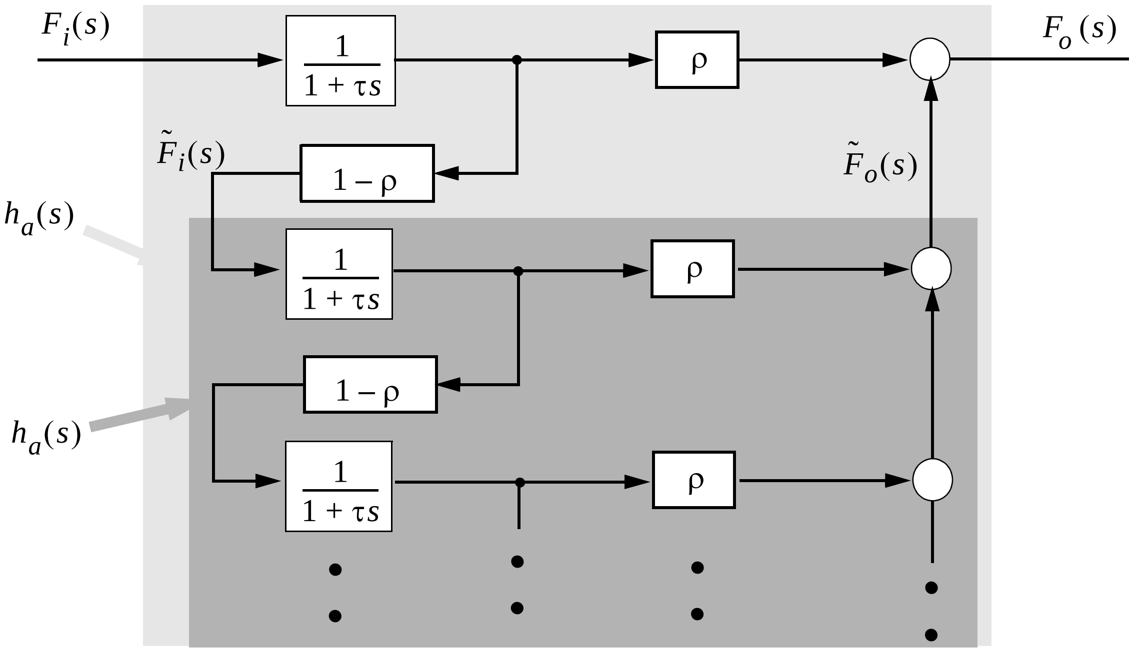

Figure 8

If we now extract the upper single unit from the aggregate, and assume that the remaining number of units is so large that this does not significantly affect the transfer function of the aggregate, then $h_a(s)$ will also be found as indicated by the dark shaded area,

$$h_a(s) = \tilde{F}_o(s)/\tilde{F}_i(s) \tag{18}$$

Employing rules for manipulating block diagrams where blocks are in parallell and in series, we get

$$h_a(s) = \frac{\rho}{1 + \tau s} + \frac{1}{1 + \tau s}(1 - \rho)h_a(s) \tag{19}$$

Solving for $h_a(s)$, we get (16). This completes the proof.

A more comprehensive treatment of this theorem and its ramifications, is given in (Andresen, 1998).

Note that this aggregation theorem makes a stronger case for the choice of a first order time lag ("vessel") model of a macroeconomy. Phillips (1954, pp. 291 - 292) chooses this model because it is the simplest one among many that has the property of a gradual response to a sudden change in the input. A similar model and reasoning is found in Godley and Cripps (1983).

Phillips (1957) discusses whether his first order time lag model from 1954 is too simple. But the above theorem strengthens the case for the 1954 model, since it is derived from the fact that an economic aggegate is a network of interacting units. Monte Carlo simulations of networks with up to 150 interacting units with randomly selected individual time lags (around a mean), and with randomly selected coefficients for flows between them, is done in Andresen (1998). These confirm that the time lag representation is a a fair approximation for the aggregate, even when the variance around the mean for generated parameters is chosen quite large, for instance unit time lags that may vary by a factor of ten.

3. A “textbook economy” with firms and households

We will consider an economy with households, firms, no government and no financial sector. Consider the diagram in figure 9: This a “physical flow chart” representation of this economy. Further below we will introduce the mathematical block diagram of the same system. As is clear from figure 9, we assume that there are no external sinks or sources of money. This assumption will be rescinded later on, among other things to discuss the multiplier.

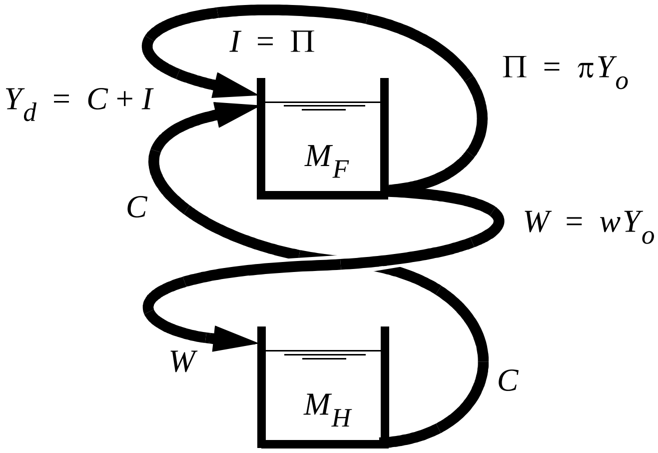

Figure 9

In the figure we have these flows:

Y_d	=	aggregate demand [currency unit / time unit]
Y_o	=	aggregate output [c.u./ t.u.]
Π	=	profit = I [c.u./ t.u.], i.e. all profits are invested, and there is no external source of investment at this stage.
W	=	wages = C [c.u./ t.u.], i.e. all wages are consumed, and there is no household savings sink (or borrowing source) at this stage.

Furthermore, w and $\pi = 1 - w$ are wages and profit share of output, respectively. They are considered constant in this model. We also have money stock in the two sectors M_F and M_H.

The mathematical block diagram of this system is shown in figure 10:

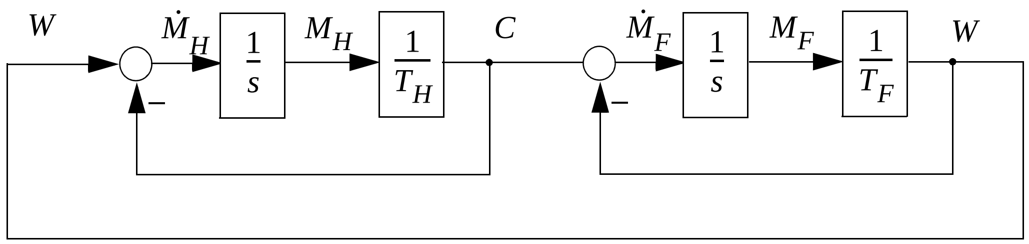

Figure 10

If we reduce the two inner loop subdiagrams, we get figure 11:

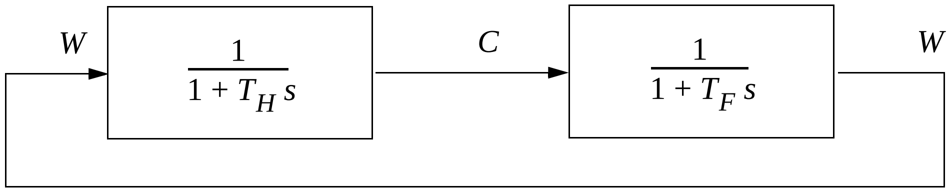

Figure 11

There is one important loop lacking in this block diagram, the profit = investment loop depicted in figure 9.We should, however, note that figure 11 is entirely correct in the sense that in a system defined as consisting of firms and households, the input to the firm sector is consumption only, and the output is wages only. Investment is a flow that is *internal* to the aggregate of firms as a whole. So how do we introduce profits, investment (and aggregate demand/output) into the block diagram representation? We do this by demanding that the two firm sector block diagrams shown in figure 12 are equivalent in an input-output sense:

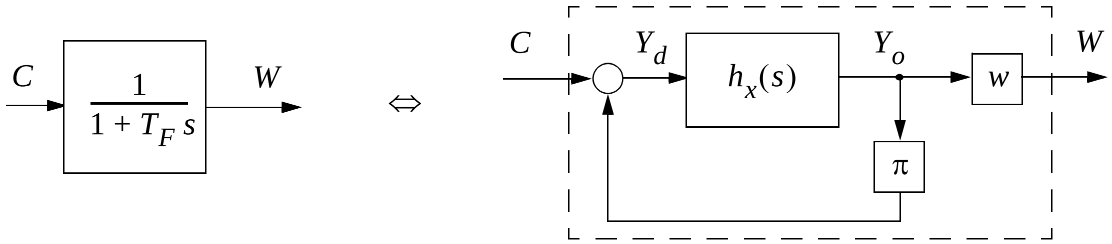

Figure 12

This gives an equation to find the unknown transfer function $h_x(s)$:

$$\frac{W}{C} = \frac{1}{1 + T_F s} = \frac{h_x w}{1 - \pi h_x} \tag{20}$$

Solving for h_x gives

$$h_x(s) = \frac{1}{1 + wT_F s} \tag{21}$$

We observe that “extracting” the profit/investment loop leads to a reduced time lag for the modified firm sector. Figure 11 may now be transformed into the equivalent block diagram shown in figure 13:

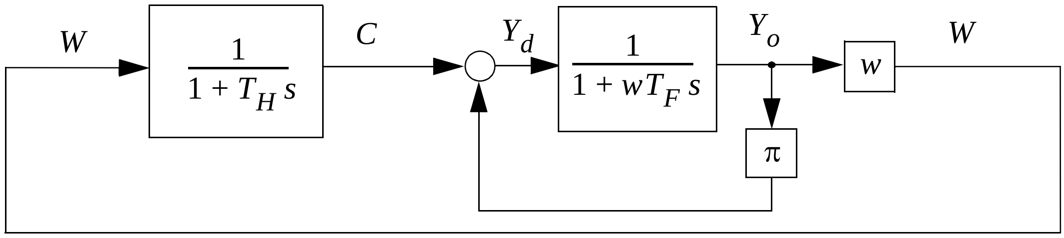

Figure 13

We may also expand figure 13 into an elementary block diagram corresponding to figure 10. The result is given in figure 14. (Note that we have here also substituted $\pi = 1 - w$.)

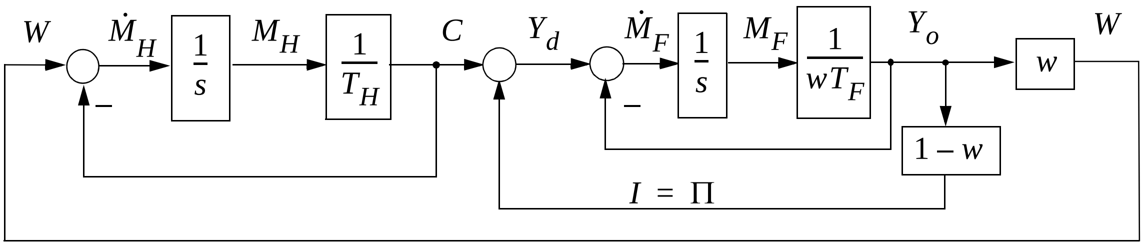

Figure 14

We have two integrators in this system. In other words, we have a system with two states; household and firm money stock. This system is autonomous (i.e. no exogenous inputs), and its time path is therefore decided solely by the initial distribution of the money stock between the two sectors. We will now use this example to illustrate the use of a modern simulation package, Simulink – and to find the equilibrium state of this system. The response of the system is shown in figure 15. Initial values are assumed to be $M_{H0} = 1400$ and $M_{F0} = 800$. System parameters are time lags $T_H = 2$ and $T_F = 20$ (weeks), and wages share of output is $w = 0.7$.

A Simulink block diagram corresponding to the one in figure 14 is shown in figure 15. This setup gives the responses shown in figure 16. (Note that syntax is somewhat different: Summation points are symbolized with rectangles with plus and minus signs, as opposed to circles used in the diagrams elsewhere in this paper.) We note how supply adjusts to demand in equilibrium. The plots also indicate that in equilibrium, money stocks are proportional to the respective time lags in the two sectors. This is easily seen by considering figure 10: in equilibrium we must have $C = W$. Since $W = M_F / T_F$ and $C = M_H / T_H$, this relationship follows.

The point of this paper, however, is to focus not on equilibrium, but dynamics. In this simple case we can find the algebraic solution for the system time path, which is

$$\begin{aligned} M_F(t) &= M_{F0}e^{-\alpha t} + \frac{M}{\alpha T_H}(1 - e^{-\alpha t}) \\ M_H(t) &= M_{H0}e^{-\alpha t} + \frac{M}{\alpha T_F}(1 - e^{-\alpha t}) \end{aligned} \quad , \quad \text{where } \alpha = \frac{T_F + T_H}{T_F T_H} \tag{22}$$

Note that total money stock, *M*, is invariant, since there are no sources or sinks of money in this model.

The system is linear and therefore amenable to algebraic solution. In a more realistic model with non-linearities, algebraic solutions are very difficult to find, if they exist at all. In such cases, numerical simulation packages like Simulink are very useful.

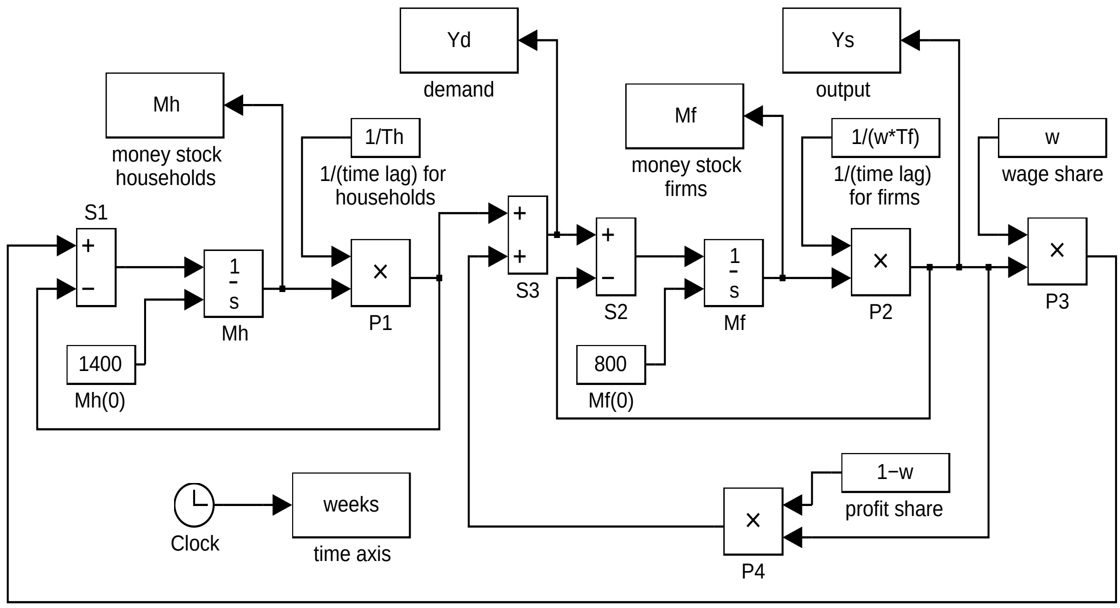

Figure 15

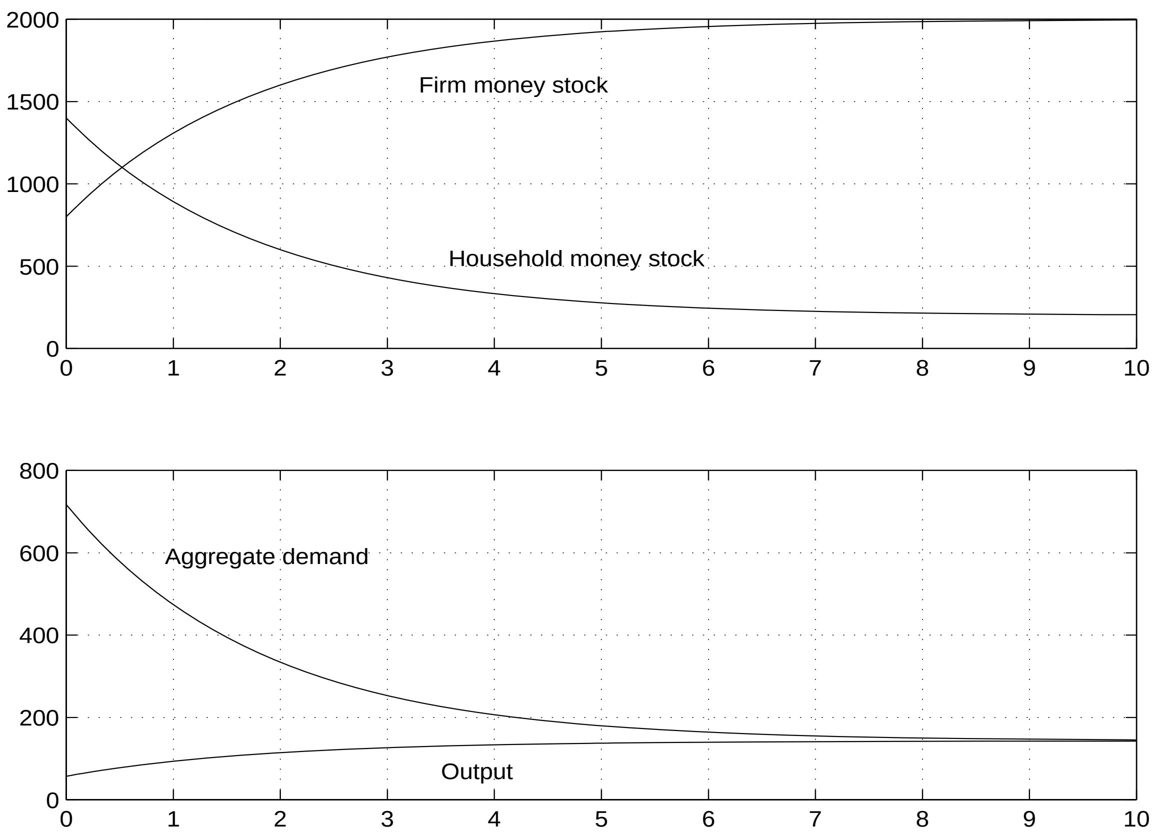

Figure 16

We will now introduce exogenous inputs to discuss the phenomenon of the multiplier. We assume the usual textbook model where all profits are paid to households together with wages, and where households consume a share *c* of their income and save the rest. See figure 17.

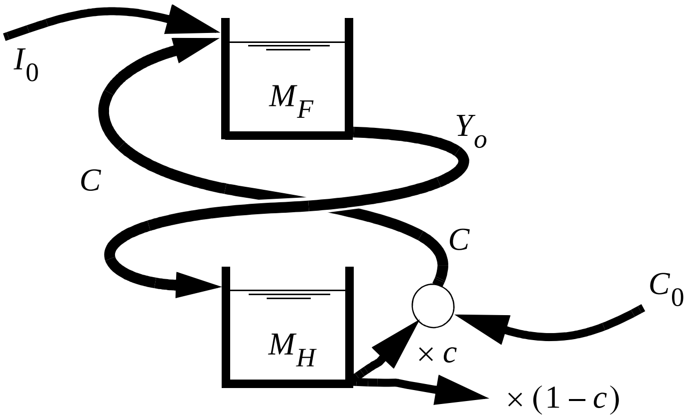

Figure 17

We have two (exogenous) input money flows, I_0 and C_0. The block diagram for the system corresponding to figure 17 is shown in figure 18.

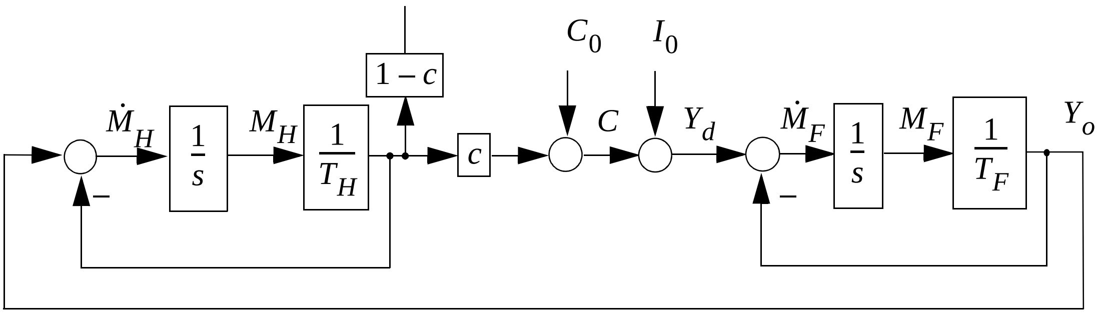

Figure 18

We see that a change in exogenous investment or consumption has the same effect. The transfer function from investment (or exogenous consumption) to output is, reducing the block diagram,

$$\frac{Y_o}{I_0}(s) = \frac{1 + T_H s}{T_F T_H s^2 + (T_F + T_H)s + 1 - c} \tag{23}$$

If we assume that investment changes as a step function with amplitude ΔI_0 at time $t = 0$, the Laplace transform of this step function is $\Delta I_0 / s$. We then have for the change in output,

$$\Delta Y_o(s) = \frac{1 + T_H s}{T_F T_H s^2 + (T_F + T_H)s + 1 - c} \cdot \frac{\Delta I_0}{s} \tag{24}$$

The final value theorem for Laplace transforms says that, for a time-dependent function $f(t)$ tending to a constant value as $t \to \infty$, we have

$$\lim_{t \to \infty} f(t) = \lim_{s \to o} sf(s) \tag{25}$$

Applying this to (24), we get

$$\lim_{t \to \infty} \Delta Y_s(t) = \lim_{s \to o} s[\Delta Y_s(s)] = \frac{\Delta I_0}{1 - c} \quad (26)$$

which is the familiar expression for the multiplier. If all income is consumed ($c = 1$), the multiplier is infinite. The system is on the border of stability: One of two eigenvalues for the system (equivalently: poles in the transfer function) is in origo. Outside sustained injection of money wil increase circulation persistently between the two sectors, since no money is taken out of circulation by households saving part of income – output increase will never stop.

The final value theorem is a fast and convenient tool to find equilibrium outcomes (if any) for linear systems, but tells nothing about the transient (i.e. before equilibrium is reached) behavior of the system. We do not bring the algebraic solution here, but instead show the time path from a Simulink run, in figure 19. The system is initially in equilibrium when investment money flow is increased as a step function by $\Delta I_0 = 5$ at $t = 25$. The propensity to consume is assumed to be $c = 0{,}75$, i.e. we have a multiplier of 4.

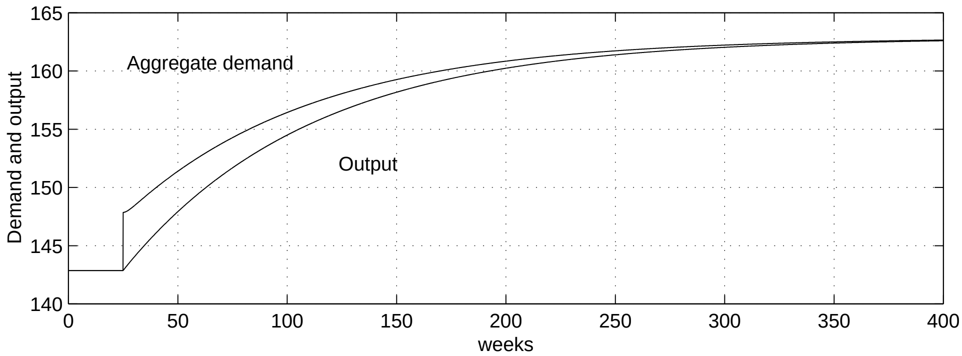

Figure 19

We observe that a 5 units increase in investment flow results in output asymptotically increasing by 20 units. Note the fairly long time lag of adjustment, which is ≈ 86 weeks.

4. The money stock inconsistency of the IS/LM model

Up to this point, our purpose has been to show how system-theoretic, block diagram-type tools are useful for macroeconomics, and to justify the first order time lag ("vessel") model as a main component in such models. We will now use this and the obvious dynamic extension of the static IS/LM model to demonstrate that IS/LM as such is fundamentally flawed. We are aware of the existence of other severe critiques of IS/LM. But the point here is that the brief analysis given below is sufficient in itself to completely invalidate it. It is not based on arguments and considerations that may be more or less convincing depending on which economics camp one identifies with – but simply on a gross mathematical inconsistency, which if true cannot be contested.

We start with the static IS/LM equilibrium equations, where aggregate demand must equal output, $Y_d = Y_o = Y$; and demand for money L must equal money stock M.

$$Y = C(Y) + I(r) + G_0 \tag{27}$$

$$M = L(Y, r) \tag{28}$$

We use a simple IS/LM variant, with exogenous net government spending G_0, and with investment being independent of output. This simplified choice makes no difference for the arguments to be made. The model corresponds to the one given in Ferguson and Lim (1998, pp 2 - 3). The relations for consumption, investment and liquidity demand are assumed linear in output and/or interest. Then we have

$$Y = C(Y) + I(r) + G_0 = C_0 + cY + I_0 - br + G_0 \tag{29}$$

$$M = L(Y, r) = kY - hr \tag{30}$$

Here *c, b, k, h* are constant parameters. We remind ourselves at this stage that this "comparative statics" model has as its premise that is a simplified representation; it is assumed to be the equilibrium solution to what in reality is a continuously varying dynamic system. Ferguson and Lim give the following dynamic extension of this model:

$$\dot{Y}_o = \alpha(Y_d - Y_o) = \alpha(C_0 + cY_o + I_0 - br + G_0 - Y_o) \tag{31}$$

$$\dot{r} = \beta(L - M) = \beta(kY_o - hr - M) \tag{32}$$

α, β are constant parameters. Verbally, these two differential equations say that the rate of change of output is proportional to the difference between aggregate demand and output, and that the rate of change of the interest rate is proportional to the difference between demand for liquidity and money stock[1].

The denominaton for the stock M is still [currency unit], while Y_d ,Y_o ,C_0 ,I_0 ,G_0 now get the denomination [currency unit / time unit] and become true flows – in contrast to their denomination in the comparative statics model, which is [currency unit].

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1. One could reasonably argue that the transaction demand for money in (32) should instead be kY_d, but the choice is to follow Ferguson and Lim. And this choice does not have any impact on the argument to be made.

We represent equation (31) by a block diagram, figure 20 [1].

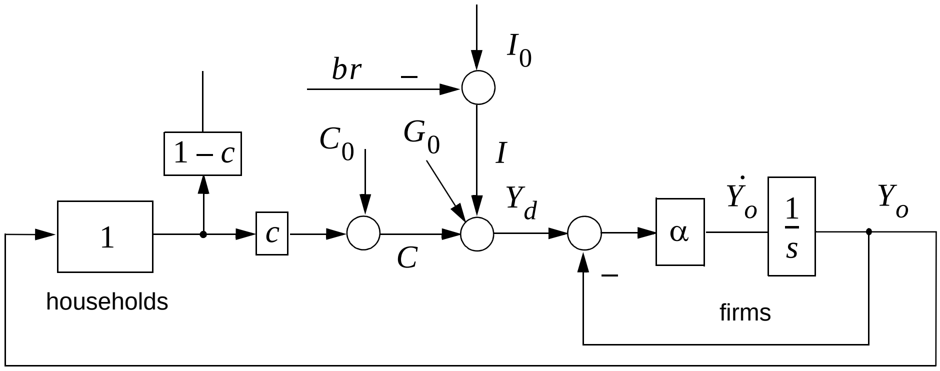

Figure 20

For the block diagram corresponding to the money market equation (32), see figure 21:

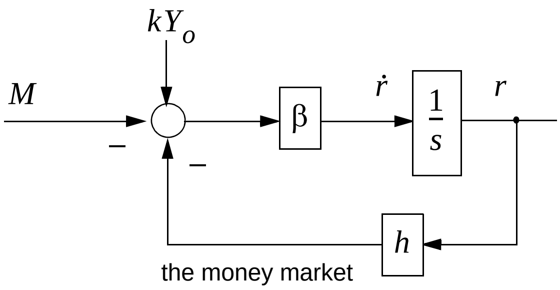

Figure 21

Before combining these two diagrams to one representing the whole system, we wish to reformulate equation (31). It may be written as

$$\dot{Y}_o / \alpha = Y_d - Y_o \text{, which must be } = \dot{M}_F \tag{33}$$

since $Y_d - Y_o$ is the net nominal money flow into the firm sector. By this we have incorporated firm money stock M_F in the model. Equation (33) explains the slightly reformulated but equivalent “firm” substructure in figure 22 below, which – except for this modification – is a result of a straightforward connection of the two sub-diagrams for the real economy and the money market.

(The modification (33) may alternatively be explained by exploiting a rule for block diagram manipulation: Interchanging the sequence of blocks on a path (in this case the two blocks containing α and $1/s$) does not change the transfer function along that path.)

1. Note that this dynamic model implies that the household sector has instantaneous dynamics, signified by the block with unity gain. Comparing with figure 18, this corresponds to the time lag in the household sector tending to zero, $T_H \to 0$. This assumption may be acceptable, since the time lag of the firm sector is so much larger due to a high share of between-firm transactions, as discussed in subsection 2.2. One should, however, be aware that this assumption implies that money stock in the household sector is zero: there is no buffer there, only a through flow.

By this modification we have accounted for the dynamics of the firm sector money stock M_F, *which in fact must be identical to the entire money stock of the economy*, since households are implicitly assumed to have no money stock, and the financial sector only appears indirectly via exogenous flows in this model.[1]

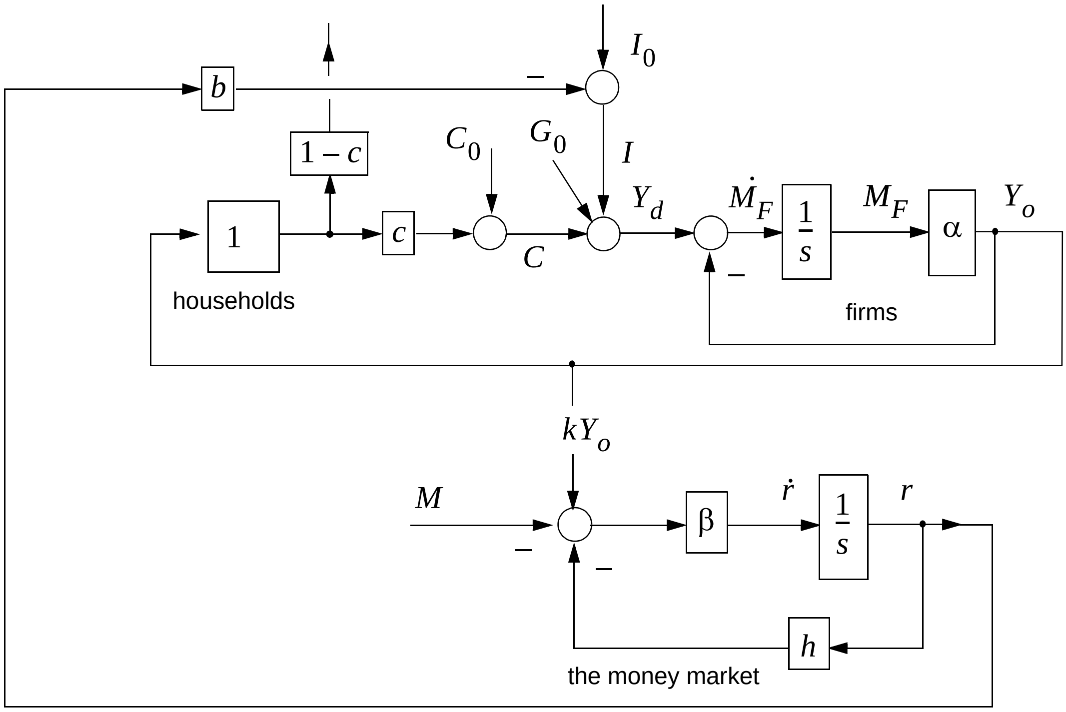

Figure 22

By now the inconsistency of the IS/LM model may be clearly observed: While money stock in reality is endogenous (M_F) and a system state, it is at the same time assumed to be an exogenous (input) variable M. What makes this inconsistency go unnoticed, is that the actual presence of money stock (M_F) within the the Y_d to Y_o dynamics, *disappears* in the (comparative) statics framework.

The correct model, in its most simplified version, should then be as shown in figure 23:

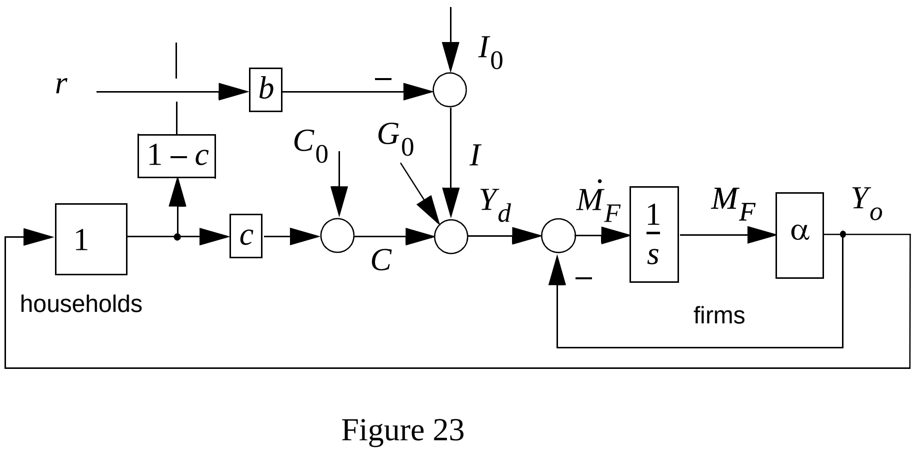

Figure 23

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1. Note how the loose ends in the model due to a lack of a financial sector stands out in the block diagram formulation. The savings flow proportional to $1 - c$ in the upper left just drains out of the system, and the flows C_0, I_0, G_0 enter the system from “somewhere”, together with money stock *M*. But this is another critique, which we don’t need to pursue here.

The model reduces to one dimension only. And *r* becomes a controlled input variable, not a system state, while *M* is no longer a controlled input variable but a system state.

5. Conclusion

If we dynamise the static IS/LM model *on the terms of its adherents* (neoclassical synthesicists), it rigorously follows that their view of money stock being an exogenous variable together with government spending (G_0), has to be substituted by the interest rate and government spending as control variables. They should then logically transit to the (Post) Keynesian position on the role of the interest rate. And all economics schools should simply abandon the IS/LM model.

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