Working paper

A countercyclical fee to eliminate long-term stock market booms and busts[1,2]

TROND ANDRESEN

Department of Engineering Cybernetics,
The Norwegian University of Science and Technology,
N-7491 Trondheim, NORWAY.

E-mail: Trond.Andresen@itk.ntnu.no

Web: http://www.itk.ntnu.no/ansatte/Andresen_Trond/ta-eng.html

ABSTRACT: A fairly simple nonlinear model that possibly contributes to understanding the mechanisms behind long-term large cycles observed in stock markets is developed. It is simulated with and without a fee in the form of a wealth tax on stocks held, the fee some per cent of current market value. The fee may be negative (a subsidy) in depressed phases. Without any fee, the model generates long-term cycles similar to those occuring today. With the fee, the cycles are obliterated (at least) in the simulations. The size of the fee is known to all at any time, continuously updated through a simple rule, with a representative stock market index as input.

1. Readers who wish to avoid tecnical parts of this paper may choose to concentrate on the “slide show” included after the appendix, on pp. 27 - 30.
2. This paper resembles an earlier version, see <http://www.itk.ntnu.no/ansatte/Andresen_Trond/econ/stockmodel.pdf>. That paper’s model has a “stock market crash mechanism”, and discusses another regulatory proposal: stocks that mature just as bonds, and are then redeemed at nominal value.

1. Introduction

There are different motivations for research into and modeling of stock market dynamics:

- To try to make a profit by using one's model as a tool for speculation.
- To use the model to gain insight into stock market dynamics.
- To use the model to gain insight into stock market dynamics and systemic weaknesses, and based on this, suggest reforms.

The first motivation seems by a large margin to be the most popular. This paper, however, stems from the third motivation. I am well aware that many market participants and (also academic) observers would react to the third motivation with the reply that such activity is uneccessary, or (having an open mind) futile since there are no interest in such reforms whatsoever in circles having the power to institute them. To this I can only reply that if there are serious problems they ought to be remedied, and one should say so even if the current probability of being ignored is high. And this type of research and discourse should be more urgent with today's free-flow and interconnected global economy, where the effects of a large event in for instance the US stock market propagates round the world in a fraction of a day, feeds back to the place of origin, and through such a process may initiate serious financial crises.

A much-discussed suggestion for stock market (and currency trade) stabilisation is to introduce a transaction tax (fee). But some empirical studies show that volatility[3] is not reduced when transaction taxs are implemented – a frequently-quoted paper is Jones and Seguin (1997). Some skeptics argue that transaction taxs are harmful, since they reduce liquidity in the market (Davidson, 1998). While being agnostic[4] on whether such

3. 'Volatility' is in this paper equated with the variance of stock price/dividend-ratio.

4. This agnosticism is based on the belief that socio-economic processes are so complex that the only way to decide issues like this is to try out proposals – just as one in the physical sciences has to do laboratory experiments to decide controversies that cannot be solved through theoretical research and discourse only.

transaction taxes will have the desired stabilising effects , I will argue that volatility as such is not the main problem. Instead two other phenomena which are related – but not equivalent – to volatility, are focused: instability, and gross long-term overvaluation. It will furthermore be argued that instability is only a problem in connection with gross long-term overvaluation. A fee on stocks held (note: not a transaction fee) is suggested to remedy this.

The paper is organised in three main sections: Short-term and long-term dynamics, a reform proposal, and finally an appendix with details about how the model has been developed and how parameter values have been decided. Some slides are appended after the appendix.

2. Short-term dynamics

The short-term model

Consider the block diagram in figure 1:

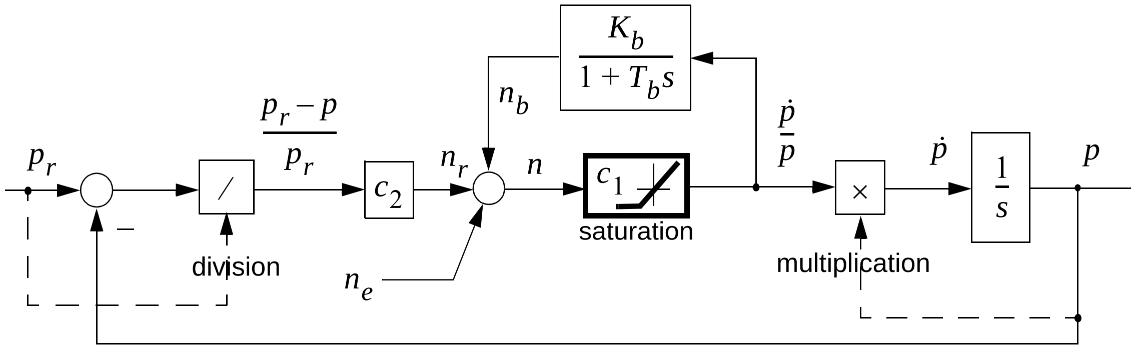


Figure 1

The symbols in the diagram are defined as follows (denomination is shown in brackets [], empty brackets mean that the corresponding entity is dimensionless):

p_r = 'real' or 'sustainable' value of the stock [ ], expressed by the price/dividend ratio it can yield in the long run. At p_r the stock is neither over- nor undervalued. For convenience I will use the term 'price' or 'value' in the following, even if I am talking about the price/dividend ratio. p_r is assumed constant in the following.

p = current market price (that is, price/dividend ratio) of stock [ ].

$\frac{\dot{p}}{p}$ = price change rate [day $^{-1}$]. The dot implies differentiation with respect to time.

s = differentiation operator [day $^{-1}$]. See footnote 5.

n = net current aggregate demand for stock [number of units]. This demand may be negative, that is, when there is a net surplus of stocks offered.

c_1 = constant factor [1 / (number of units $\cdot$ day)] transforming net demand into price increase rate. The total number of stocks issued is incorporated in this factor. There is, as indicated in the figure, saturation in price decrease rate, since current surplus offered cannot exceed the total number of stocks issued. In the model, this is translated into a maximum rate of price decrease. This saturation will only be reached in connection with panics, treated in a later section.

c_2 = constant factor [number of units] transforming price deviation into corresponding net demand. The total number of stocks issued is incorporated also in this factor.

Surplus aggregate demand is now assumed to consist of three components,

$$n = n_r + n_b + n_e \tag{1}$$

We will from now on use the term ‘demand’ in the sense of ‘surplus aggregate demand’. We have

n_r = Component due to ‘fundamentalist’ agents’ belief about the sustainable value of the stock.

n_b = Component due to agents watching price increase/decrease rate and doing ‘technical trading’ based on this. The sustainable value of the stock has no direct influence on this component. Subscript *b* signifies ‘bandwagon’.

n_e = Component due to fundamentalist agents having different estimates of the sustainable value of the stock, and to bandwagon agents taking differing action based on the same price increase rate. This component also incorporates modeling errors and simplifications (see below). n_e is assumed to be a zero mean stochastic process. Subscript *e* signifies ‘error’.

The differential equation

$$T_b \frac{dn_b}{dt} = -n_b + K_b\left(\frac{\dot{p}}{p}\right) \tag{2}$$

which corresponds to the transfer function[5] $h_b(s)$ from $\dot{p}/p$ to n_b, shown in the block diagram in figure 1,

$$h_b(s) = \frac{K_b}{1 + T_b s} \tag{3}$$

is chosen as a model of the speculative component of demand: There is a positive feedback through $h_b(s)$ from price increase rate to the demand component n_b. If for instance $\dot{p}/p$ is large and positive at a certain moment, many agents will jump on the bandwagon and buy now in the hope that prices will continue to rise. Of course some technical trading strategies are more elaborate than this, for instance action in 'counter-phase', buying when prices are falling in the expectation that they will rise later on. It is assumed however, that herd mentality is the dominant type of speculative behaviour. Invoking Occam's razor, the simplest model that accounts for this is given by equations (2), (3). The parameter T_b expresses the small time lag from acquiring price information to buying (or selling), that speculative action cannot get around. This lag is due to delays in acquiring information, considerations, and then getting the trading done. The gain K_b expresses how strong speculative action is, based on the available price change rate information.

Note the term 'action', as opposed to 'agents'. Individual agents may of course operate in a purely speculative or herd mode, others may again be pure 'real investors'. But most have composite motives (real-economic more or less off the mark, and speculative). When the market as a whole is considered, however, this discussion becomes unimportant, since the market as a whole must neccessarily have a 'composite motive'.

The surplus demand component n_e accounts for the aggregate effect of agents making erroneous and different assumptions about the stock value, but in such a way that the mean surplus demand error is assumed to be zero. We also incorporate the effects of differences in individual speculative behaviour into this noise process, since in reality each speculative agent will act according to unique dynamics, which will also be nonlinear with

5. Here s is a *differentiation operator*, so that $y(t) = \frac{1}{1 + \tau s} x(t) \Leftrightarrow (1 + \tau s) y(t) = x(t)$, shall be interpreted as $y(t) + \tau \frac{dy}{dt} = x(t)$; a linear differential equation with input x and output y.

parameters and structure that will change with time. All this individual behaviour is averaged into the linear, time invariant transfer function (3). What is lost through this simplification is then assumed to be to a sufficient degree represented through the error process n_e. Thus, this process has at least two components: Erroneous estimates of the stock's sustainable value, and modeling errors due to aggregation of the trend chasing ('bandwagon') feedback path. A final and third component is the effect of different exogenous economic news that influence the valuation of the stock.

If we now consider a situation where the price of the stock is at its sustainable value, the market should have no real-economic incentive to trade. But trading will take place all the same, since exogenous changes influence the market – this is the 'news' component of the process n_e. And individual more or less rational, more or less well-informed agents have their own differing assessments, and trade based on this, even when there are no exogenous 'news'-related impulses. In the system-theoretic language of this paper, we may say that the error process n_e is a 'disturbance' that excites the system, so that the market is never in equilibrium, but fluctuates around it.

For small fluctuations Δp around a constant p_r, so that $p = p_r + \Delta p$, we have a linear system which is excited by the error process n_e. The transfer function from n_e to Δp is

$$h_{p,\,n_e}(s) = \left(\frac{p_r c_1}{T_b}\right) \frac{1 + T_b s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}, \tag{4}$$

where the undamped resonance frequency is $$\omega_0 = \sqrt{\frac{c_1 c_2}{T_b}}, \tag{5}$$

and the relative damping factor is $$\zeta = \frac{1 + c_1 c_2 T_b - c_1 K_b}{2\sqrt{c_1 c_2 T_b}} \tag{6}$$

The two eigenvalues of the system are indicated as small dots in figure 2:

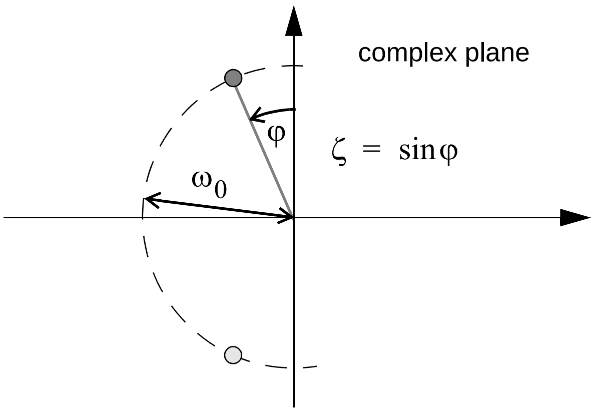


Figure 2

Consider a case where K_b is increased while T_b is held fixed, that is, speculative action is stronger, while the information/decision time lag remains the same. From (5) we see that ω_0 is independent of K_b, while (6) implies that ζ decreases with increasing K_b. In terms of figure 3, this means that the eigenvalues of the system move along the circle towards the imaginary axis.

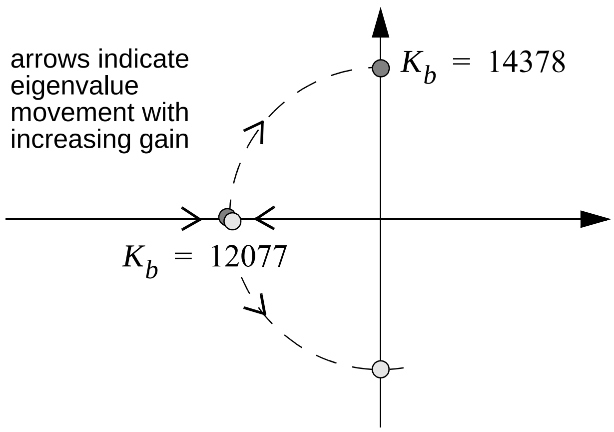


Figure 3

The system approaches the border of instability, which means increased volatility: For a given variance in the error process n_e, the variance in price will increase with K_b. This follows from the two expressions

$$\phi_{pp}(\omega) = \left|h_{p,n_e}(s)\big|_{s=j\omega}\right|^2 \phi_{ee}(\omega) \tag{7}$$

$$\mathrm{var}(\Delta p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \phi_{pp}(\omega)d\omega, \tag{8}$$

where $\phi_{ee}(\omega)$ and $\phi_{pp}(\omega)$ are the power spectral density functions for the input noise process n_e and the output zero-mean process Δp, respectively. If h_{p, n_e} is close to unstable, it has a high resonance peak near frequency ω_0, which is then mirrored in $\phi_{pp}(\omega)$, following (7). This translates into a large area under $\phi_{pp}(\omega)$ and a correspondingly high $\text{var}(\Delta p)$, according to (8).

There is an interesting insight that emerges from the model at this stage. Again consider figures 2 and 3. Numerical values for K_b, and all other parameters have been chosen through a procedure described in the appendix. The eigenvalues are located on the imaginary axis ($\zeta = 0$) and the system is on the edge of instability, for $K_{b1} = 14378$. On the other hand the system is *overdamped* (i.e. 'non-volatile') for $\zeta > 1$, which we have when $K_b < 12077 = 0.84K_{b1}$. Since real stock markets seem to be (see appendix) *underdamped* ($0 < \zeta < 1$), this suggests that there is some adaptive mechanism at work to keep K_b close to the gain that makes the system unstable.

That mechanism may be the following: One one hand, agents are attracted by short-term movements, to exploit them through trading. By their participation, they increase K_b and volatility, by this attracting even more trading. But when the market gets too turbulent, some of them abstain, thus decreasing K_b, and temporarily stabilising the system. This type of behaviour is confirmed by prominent Norwegian speculative traders in an interview (Aftenposten report, 2000).

The resulting fluctuations between system stability and instability also explain the irregular bursts of volatility that characterise time series of stock prices. This issue is discussed in Lux and Marchesi (2000), who attribute such bursts to fluctuations in the relative strength of bandwagon versus fundamentalist populations. For the purposes of this paper, we will see that it is not neccessary to choose between these explanations, or to account for the burst phenomenon. It suffices to model short-term dynamics as time invariant, but such that the system is close to instability.

The ideal stock market

The textbook argument for a stock market is that it is an optimal way to channel surplus money into investment: The sum of all participants' trading activity channels society's surplus into such enterprises as are considered by the collective mind of the market to have the best future prospects. If we relate this conception of a stock market to the model in figure 1, this corresponds to the case $K_b = 0$. All action is then taken on the basis of each agent's best valuation of a firm's prospects, regardless of what other agents do. Demand is

$$n = n_r + n_e, \tag{9}$$

there is no 'bandwagon' component n_b. The transfer function (4) from n_e to Δp now reduces to the first-order expression

$$h_{p,\,n_e}(s) = \frac{c_1}{s + c_1 c_2} \tag{10}$$

If we use this in (7) and (8), we find that this market will have much lower volatility than for the case $K_b > 0$. More important, the system is now dramatically more stable. A smaller value of the inner loop gain $K_b c_1$ has the advantage of making stock market crashes much less probable. A possible stabilisation solution is therefore some institutional market reform that reduces K_b without increasing c_1. This is, however, outside the scope of this paper, where the parimary concern is not instability and crashes, but eliminating long-term gross overvaluation – long term "bubbles". If such processes are nipped in the bud crashes will not happen, because a grossly overvalued price is a neccessary prerequisite for a crash to occur. Thus the positive feedback and instability due to short-term heard mentality is not dangerous any more, even if it still gives rise to what many would say is "excess" volatily.

3. Long-term dynamics

From now on let us consider the model not to represent a specific stock, but a 'composite stock', composed of stocks from all firms listed on a stock exchange. The composite stock p/d ratio is roughly proportional to a deflated stock exchange index. For short-term movements inflation may be ignored. The p/d ratio for the 'composite stock' is defined as the total value of all stocks traded on the exchange, divided by the total sum of dividends. All stocks are assumed to have roughly similar dynamics. These assumptions mean that the composite stock p/d ratio (from now on for convenience called the 'index' or 'price') will also fluctuate around p_r, with dynamics that are similar to those for one specific stock. The difference is that a price shock for one stock only, does not impact very strongly on the index (as opposed to for instance a Central Bank interest rate change, see examples in appendix). The aggregation step from one stock to an index is comparable to the earlier step of aggregating all agents into one composite agent. We uphold all variable and parameter names and numerical values introduced for the stock model, with the note that they now pertain to the index. By now we are ready to consider an augmented model as shown in figure 4 (next page). Again we assume that model imperfections and approximations, and fluctuations in demand for the 'composite stock' implied by the index, is accounted for by a zero mean noise process n_e as introduced earlier. We recognise the short-term model in the lower half of the figure. Ignore until further notice the nonlinear function outlined in bold in the upper part of figure 4; assume this to be unity for the time being. Consider the block in the upper right-hand corner. The instantaneous price increase rate is an input to the block, which is a low pass filter with time lag T_f of a one year magnitude. The rationale for this filter is that the increase rate of 'optimism' (or 'confidence', 'bullishness', 'animal spirits') is assumed proportional to the long-term trend in index increase. Hourly, daily, even weekly and (to some degree) monthly fluctuations are disregarded; there is a sluggishness in market mood. Price appreciation has to be persistent over a long time before the market really picks up. On the other hand, when the price culminates and starts falling, the market will need a corresponding amount of time for such a change of affairs to sink in. The increase rate in optimism is set equal to the market's perception of the long-term index increase rate. Optimism is given a numerical value, and a range which is both positive and negative. Thus 'pessimism' corresponds to negative optimism.

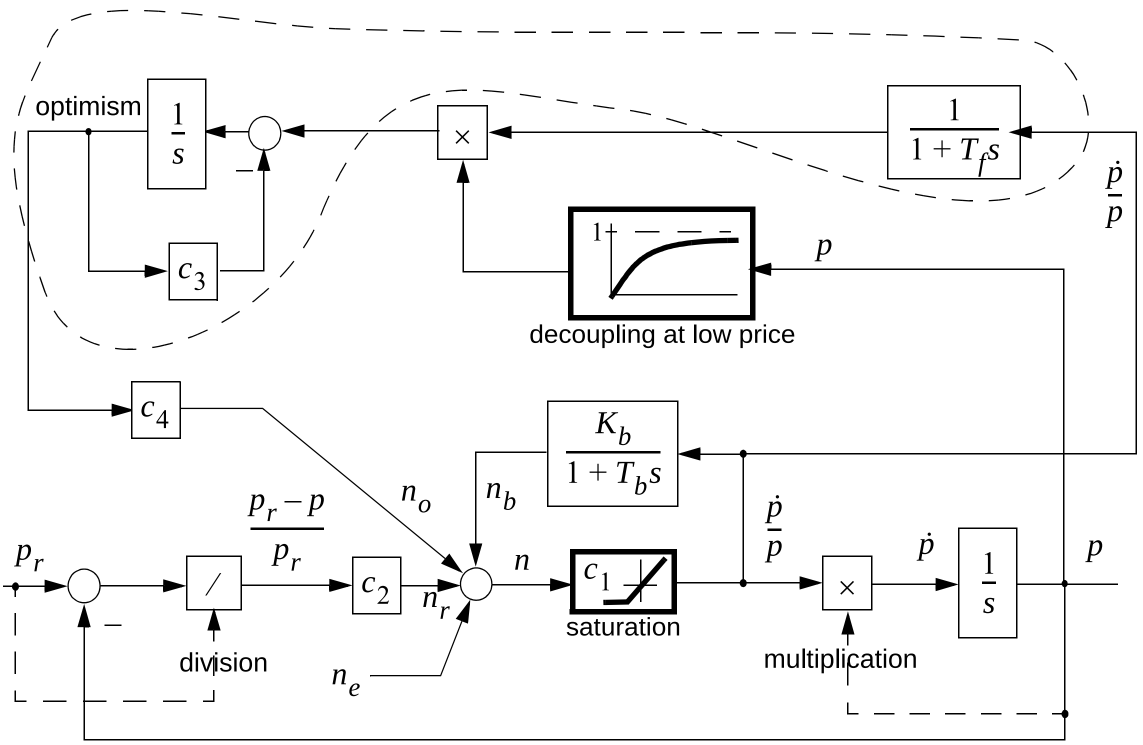


Figure 4

By now it should be clear why it has been neccessary to transit from an invidual category of stock to an index: Market mood is a function of the behaviour of the aggregate of all stocks, not one category only.

In the absence of any perception of a long-term tendency for price to change (that is, a flat price level over a long perod), the current level of optimism will slowly erode to zero, through the "forgetting factor" c_3. The argument for this is that the market will gradually forget its initial mood and tend towards a neutral attitude (zero optimism or pessimism) if the mood is not maintained by a sustained increase or decrease in stock price.

Let us see what happens if we isolate the upper subsystem encircled by a dotted line from the model, input a rectangular price increase rate pulse, and observe the response in optimism. We assume a one-year (defined as 250 trading days) constant price increase rate pulse. This pulse, and the corresponding response in optimism, is illustrated in figure 5.

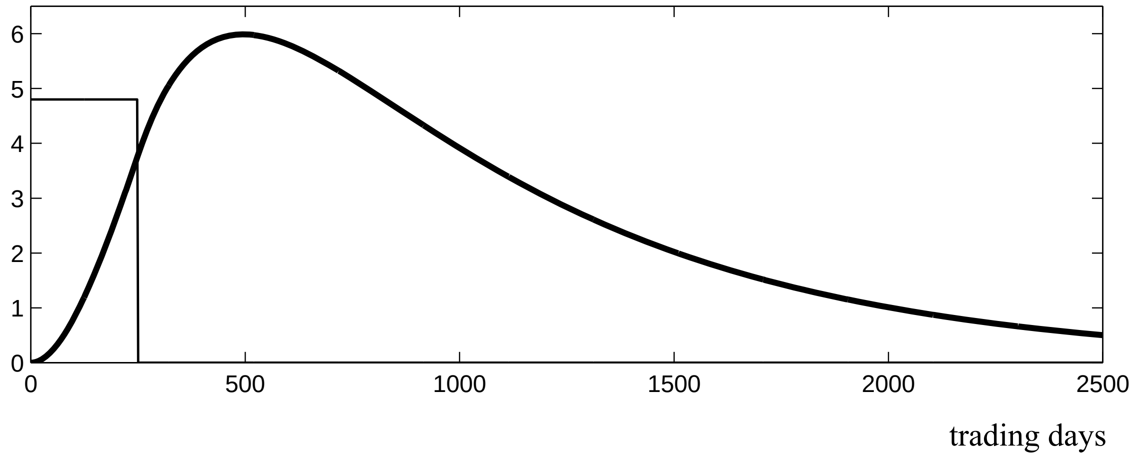


Figure 5

The input pulse is not shown to scale. The choice of parameter values is discussed in the appendix. Note how optimism culminates after around 500 trading days (2 years). There is thus a very large difference between the choice of fast dynamics of the bandwagon loop (minutes, hours), as opposed to the newly added long-term mood loop (years).

To complete the explanation of the long-term mood loop, consider the nonlinear function in figure 4, ignored until now.

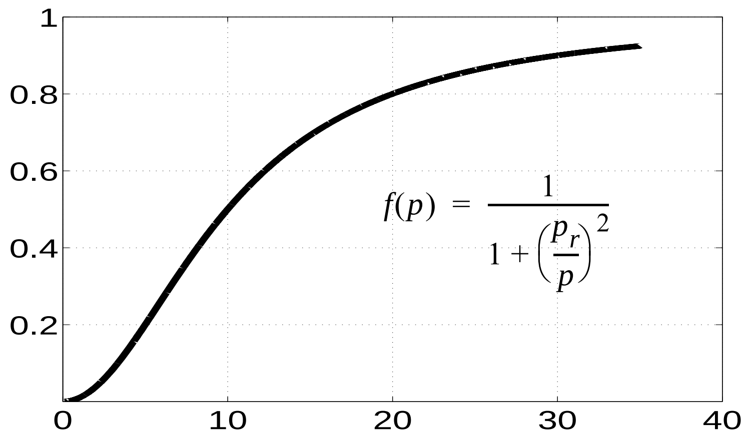


Figure 6

This function, shown in figure 6, introduces a weakening of the coupling from price change rate to mood change when the price is low.

Without this modification, simulations break down by the system “diving” at an accelerating rate into zero price during the downswing phase. This happens because when p is small and falling, the price change rate $\frac{\dot{p}}{p}$ grows strongly negative, thus accelerating the negative mood. There are at least two possible modifications to this unreasonable trait of thye model: One is to simply introduce a soft limit to mood on the negative side, another is the one chosen – to reduce the coupling from the price change rate to mood change when prices are low. This should be reasonable also in a real word sense: Agents are probably less mood sensitive to price changes when the price is low – they are holding out and waiting for better times.

We have in this subsection introduced two first-order linear blocks and one nonlinar relation, which together form the long-term mood loop. Parameter values have been decided through a simulation-based trial-and-error process described in the appendix. With the resulting choice of parameters (and no external noise exciting the system; $n_e = 0$), the cycles look like in figure 7.

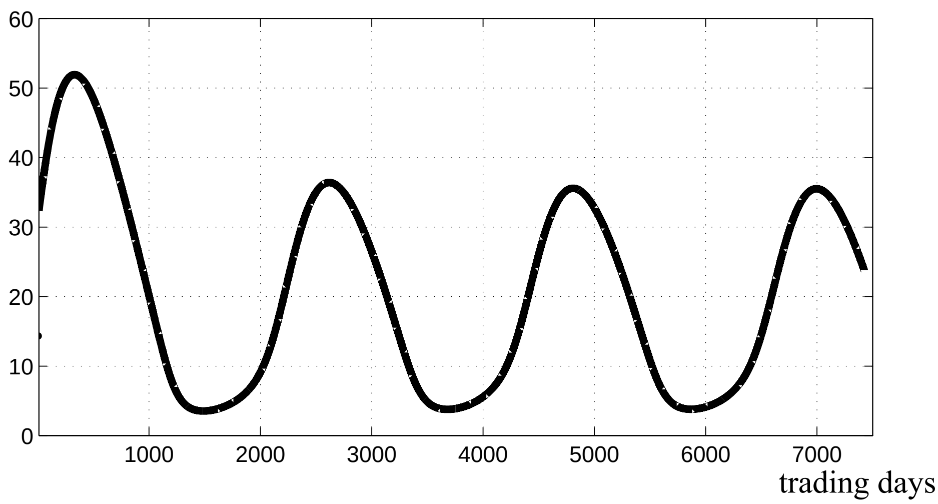


Figure 7

The first cycle is somewhat different depending of the choice of initial values. But, regardless of initial values, after a while the system settles down to the same regular (limit) cycle.

Note that the model is not dependent upon a crash-and-subsequent-recovery mechanism for cycles to occur, or upon being excited by any exogenous variable, for instance n_e. This system will always self-oscillate, and can never be in equilibrium. The dynamics stem from endogenous mechanisms whch may be verbally described as follows: The upswing is due to the spreading and self-reinforcing belief that 'if I get in now, I can always cash in my investment with a profit at some later time'. This upswing, however, sooner or later has to culminate at some level, when the feeling has spread enough that current prices have grown far too high in relation to the sustainable value of the stock, and action is based on this. In the model this is accounted for by a gradually more negative demand component n_r. This leads to a stagnating price growth trend which is reflected in slower growth of optimism, which again feeds back through demand to further slowing of the price growth rate, and the price will eventually culminate. The optimistic mood will start deteriorating when price has stopped increasing, and this gradually leads into a downswing. But the downswing also needs time to build up momentum. Years later, things will pick up again after the pessimistic mood culminates because of a sustained and now positive demand component n_r. The upswing starts, and one cycle is completed. *The period time of a cycle is in the main decided by the inertia of market perception*: It needs time to absorb a persistent tendency in price change, and it needs time to forget.

The reader may at this stage object that the downswing predicted by our model is very smooth and well-behaved. Where are the panics, which may erase a substantial part of an index in part of a trading day? Different panic mechanisms are conceivable. A panic may be triggered by an easily identifiable event (for instance a strong and unexpected Central Bank interest hike), or simply a random large price dip. But for the purposes of this paper, we will abstract from[6] panics and confine ourselves to the model in figure 4, which endogenously generates long term swings that we want to dampen.

6. Presentation of, and simulation with, a model that incorporates a "crash mechanism" is done in an earlier version of this paper, see here.
That paper also discusses another regulatory proposal: stocks that mature just as bonds, and are then redeemed at nominal value.

4. A variable “anti-bubble” fee on stocks

We have until now assumed that *p* is the aggregate price/dividend ratio corresponding to a certain stock market index. From now on we assume that *p* is simply the index itself (deflated). The model developed is assumed to give a fair representation of system dynamics also with such a definition of *p*. This change is made because regulatory action is to be taken based on some representative deflated stock market index. The idea is to levy a small fee on stocks held, the fee a few per cent per year of current market value. The fee may be negative (a subsidy) in depressed phases.

Without the fee, the model generates long-term cycles as already demonstrated. With the fee, we will see that the cycles are damped out – at least with this model. The size of the fee is known to all agents at any time, continuously updated through a simple rule, with the deflated stock market index as input.[7] We will now derive this rule, and test it out. Consider figure 8 next page, which is an augmented version of figure 4: it has a new “rule generator block”. This rule generator (or controller in control engineering terms) takes price increase rate as its input, and outputs the current fee which has the same denomination [%/year] (strictly it is %/day in the simulations, but for convenience of presentation I will use year as the time unit here). The market is assumed to function such that the fee cancels the effect of the optimism that otherwise gives rise to the cycle. For this cancellation to be correctly accounted for, the model in figure 4 has had to be slightly reformulated: The upper left subsystem is now expressed as a first-order time lag on a par with the right subsystem, with a time lag $T_o = 1/c_3$. The gain in the middle left block with the optimism-driven demand component n_o as output, has been changed from c_4 to c_4/c_3. These modifications together ensure that the dynamics of the upper long-term

7. It may alternatively be updated regularly and manually by a comittee, in a similar way that the Central Bank interest rate is set today – using the result generated by the rule as a guideline figure. But there are dangers with this, see discussion on the following pages.

loops are unchanged, but now we have achieved that the output from the optimism subsystem has the right denomination, [%/year], and it is therefore meaningful to subtract the fee directly from this output to get the remaining (ideally: zero) signal that drives the n_o component.

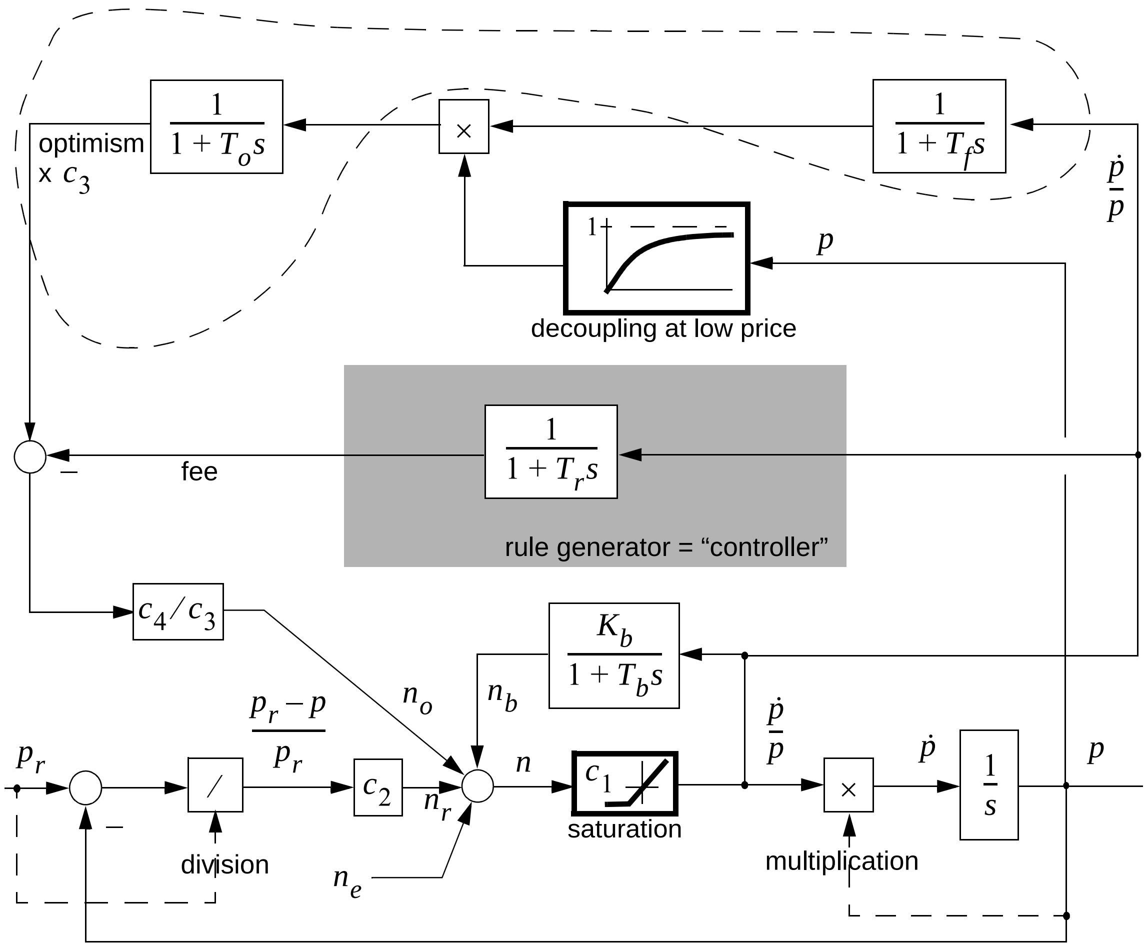


Figure 8

We assume until further notice that the nonlinear decoupling block may be considered constant = 1 (the case when price p is large). Then the series connection of the upper blocks with time lags T_f and T_o is approximately equivalent to a single linear 1. order block similar to the controller, with time lag $T_f + T_o$. Based on this, we initially choose the controller's time lag as

$$T_r = T_f + T_o \tag{11}$$

Since the two upper blocks in series may be considered equivalent to the controller (in a coarse sense), the subtracted fee should cancel most of the optimism component.

Before presenting simulations, we may make the interesting observation that T_r is a large time lag deliberately incorporated in the controller, giving it a slow response, making it very "sluggish" (the numerical value of the sum $T_f + T_o$ used in the model is 914 trading days = 3.7 years, see appendix). Sluggishness is usually not something one wants in a controller, but will be seen to be necessary for good control action in this case. The large T_r has another interesting consequence: control action will not interfere with daily, weekly, monthy or even year-long swings, so that other market dynamics and volatility than the very long-term cycles may play themselves out without being "interfered with".

Simulations

Consider figure 9: It shows the system's response with "perfect" control, which is what we have when the controller is simply a replica of the upper feedback loop.

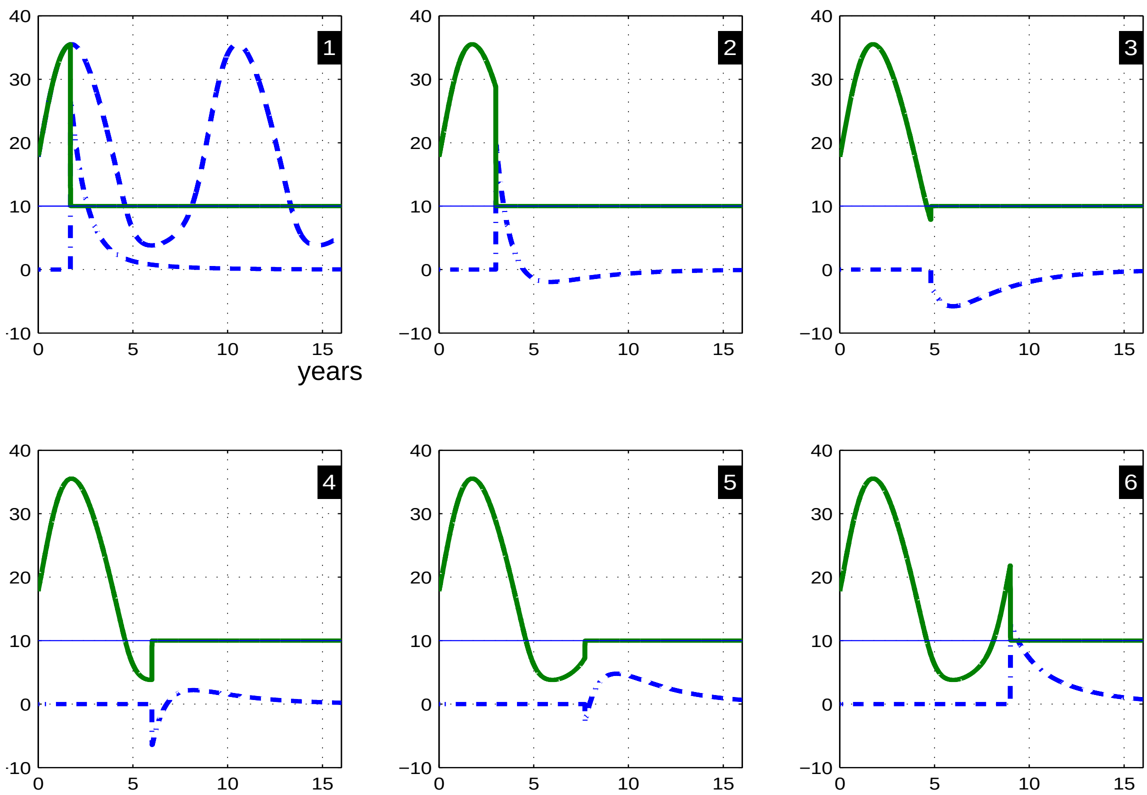


Figure 9: Perfect control – optimism component of demand is obliterated

This ideal controller will be used as a reference. It gives a fee that exactly counters the output from the upper feedback loop (the output labeled “optimism × c_3” in figure 8). Before proceeding to corresponding results given by the simpler controller in figure 8, some explanations and comments based on figure 9: In plot 9.1, the cyclical price response is shown without any control at all (dashed graph), for comparison. In all six runs, we observe the effects of the controller being activated, at different points in time. Price immediately jumps to p_r as expected. The lower graph (dashdotted) for all runs is the fee levied. We will comment on the fee graph for three of the cases:

9.1: In this case it peaks at ~24 % (more on this below), but falls fast and is close to zero three years later. As observed, it drives down the extremely overvalued, near-peak market very fast.

9.3: Note in this case how the counter-cyclical fee is negative, i.e. it is a subsidy, at max. 5% for a short time. This is to stop a market which has nose-dived past "fair value".

9.4: A subsidy of ~8 % is applied for less than a year to inject some optimism in a close to bottom phase, but later changes to a fee peaking at ~2-3 % to stop the market taking too much off.

The very high yearly fee of 24 % in the first case is due to extremely unrealistic circumstances: The controller is not activated before the market is at its absolute peak. But with the controller in continuous use, the market will fluctuate close to p_r, and fees will be correspondingly smaller, and of a reasonable size.

let us now do the same six runs with a simpler and less perfect controller, the one in figure 8. Figure 10 shows the results. Comparison with figure 9 shows that this controller as expected doesn’t perform as good. It is slower in bringing the system to p_r, but it breaks the cycle and stabilises the system. Changing the gain in the controller or its time constant does not give better results (simulations not shown here). The advantage of this controller is that it is simple and more easily understood by market participants. It may then be used as input to a fee-setting stock market comittee, which decides on the fee regularly, in a similar manner of the central bank's adjusting the interest rate. But the system would perform badly if this comittee "did not take the controller seriously", and chose the fee without understanding the need of several years' "inertia" in the process, i.e. the changing of the fee must be very gradual. This would be a case of human intervention where decision makers have difficulties grasping dynamics with long time scales.

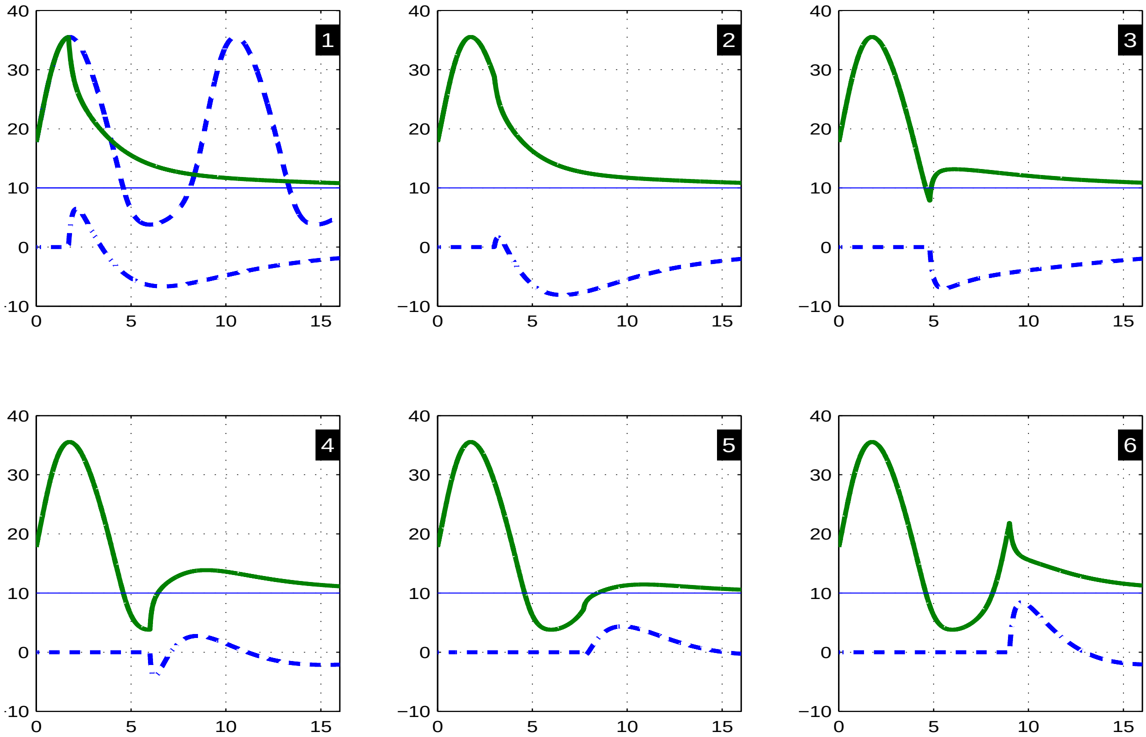


Figure 10: Control with a first-order control rule

Appendix: Building the model, choice of parameter values

The general problem with modeling the dynamics of an index, based on real-world data, is that we lack information about what sort of processes excite the system. We can measure the system output (the index), but know very little about the input. The method chosen here is therefore to search out historical events where one specific and known input is so dominant for some reasonable time window, that this together with the index behaviour for the same period can be used to extract a coarse system model that is valid for the short-term.

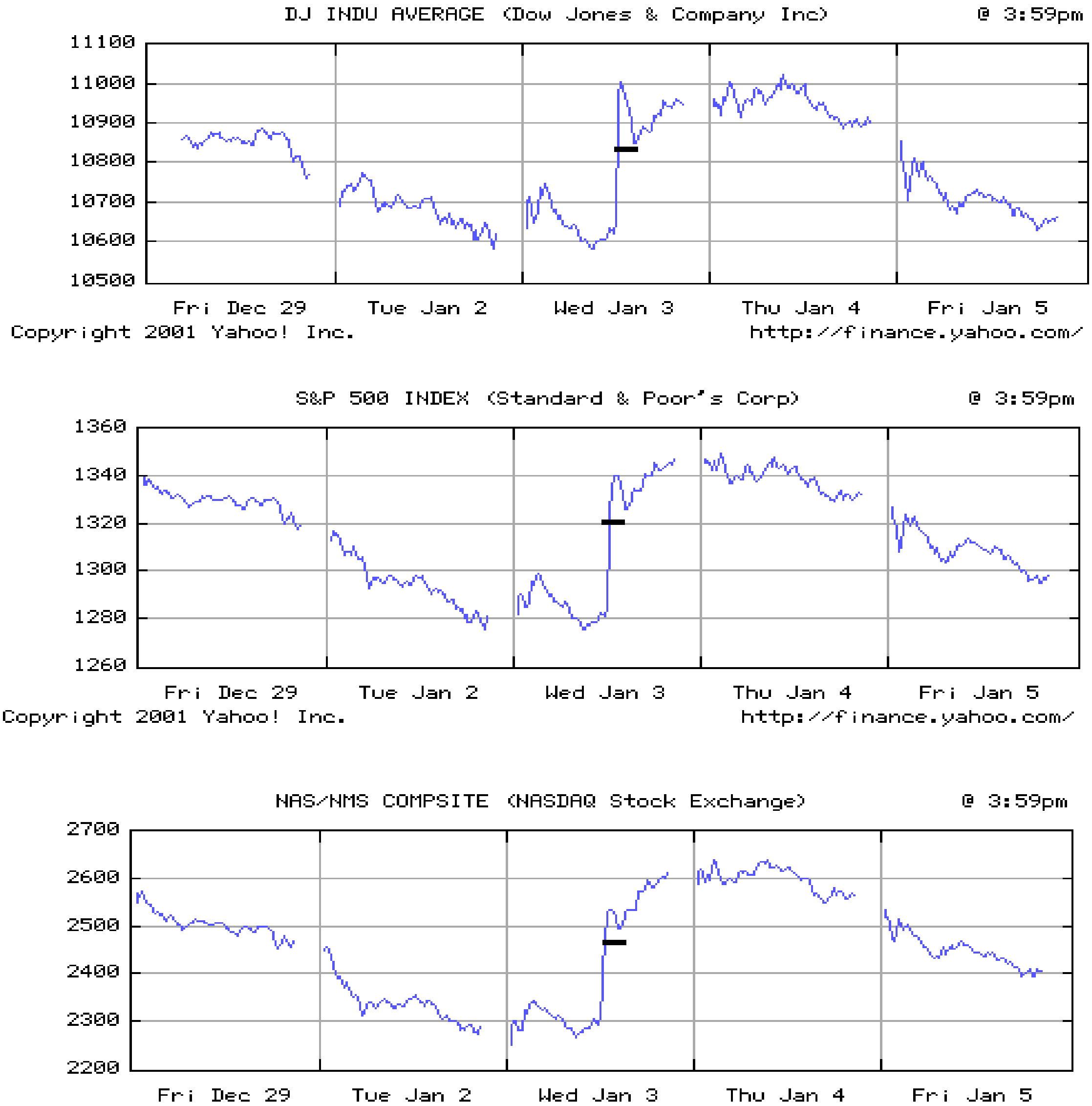


Figure 11

Such an event is a Central Bank interest decrease. Referring to figure 1, this corresponds to inputting a small positive stepwise increase to p_r (or equivalently, except for a constant multiplicative factor, to an initially zero n_e).

Figures 11 and 12 show the responses for some U.S. indices in connection with two different Fed interest rate reductions, on 3 January and 18 April 2001, respectively. (The April graphs show only the pertinent part of the trading day.) The horizontal bars corresponds to one tenth of a trading day of 6.5 hours.

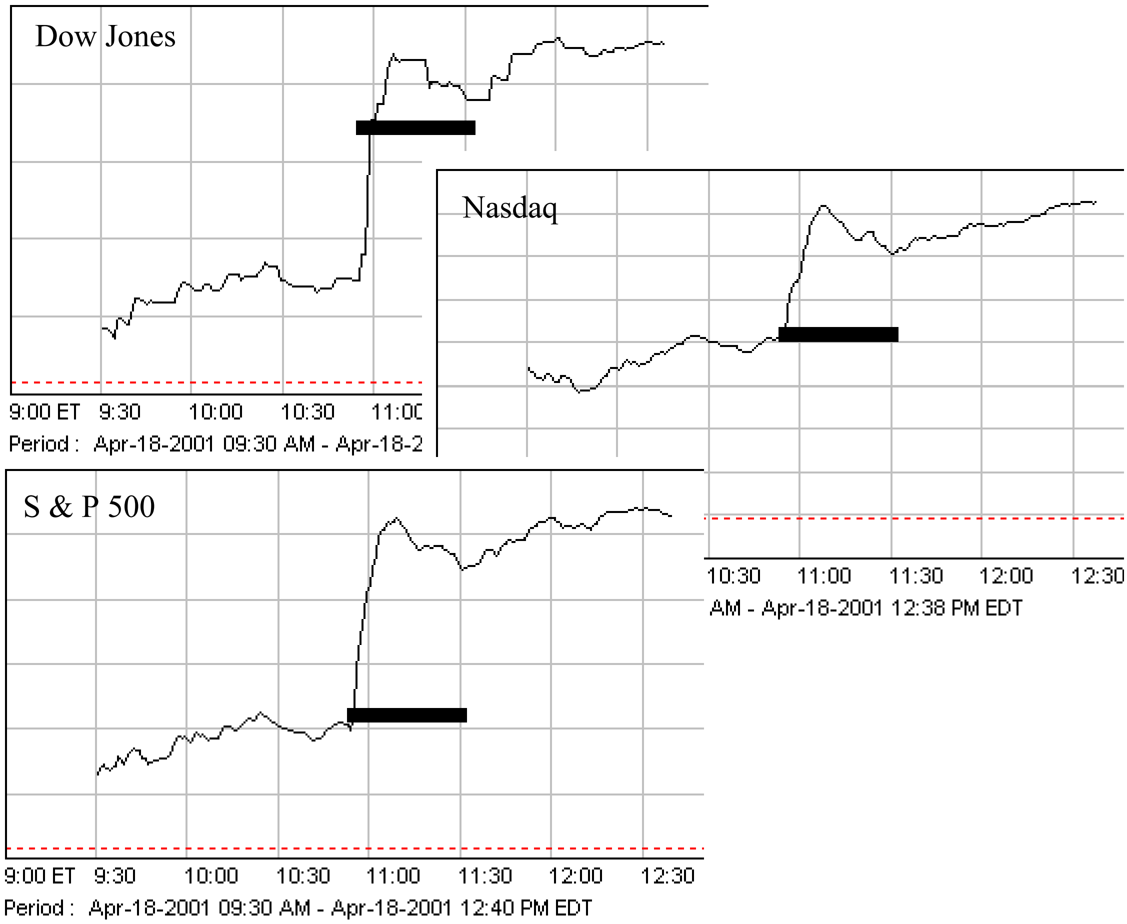


Figure 12

We note that a common feature is a steep index increase, overshooting upwards and slightly downwards before the reaction to the interest rate hike tapers out so much that it, except for the system remaining at a higher price, is drowned by the effects of other and not knowable input processes.

The simplest model to account for this type of underdamped step response is a second-order linear system, which is the one initially chosen and shown in figure 1. The system in has been simulated for a range of settling times and relative damping values. One step response that was considered acceptably close (by visual inspection) to the data in figures 11 and 12 is shown in figure 13:

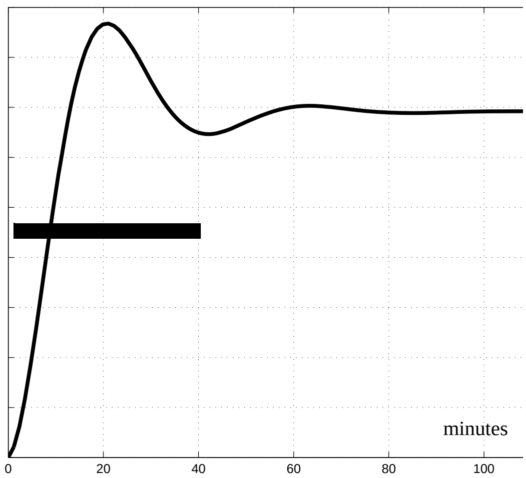


Figure 13

The system that generated this response has undamped resonance frequency

$$\omega_0 = \frac{2\pi}{T_0} = 62.8\text{, where the period time is } T_0 = 0.1 \tag{12}$$

and the relative damping factor is $\zeta = 0.4$ (13)

We now choose a reasonably short time lag for bandwagon behaviour,

$$T_b = 0.00128 \text{ [trading days] } (= \text{ half a minute }) \tag{14}$$

We may freely decide the coefficient c_1, and choose $c_1 = 7 \cdot 10^{-5}$. By this, the second.order model is uniquely determined, since c_2 and K_b may now be calculated from equations (5) and (6).

The sampling time for storing simulation results was chosen as $T = 1/13$, but the discretisation interval for numerical simulations had to be chosen smaller than T_b, and was therefore set to $T/100 = 0.6T_b = 0.3$ [minutes]. The numerical simulation algorithm chosen is the fixed-step Euler method. The ‘sustainable price’ p_r is chosen as a constant; $p_r = 10$ throughout.

At this stage, let us digress to again consider the special idealised case with $K_b = 0$, i.e. no bandwagon behaviour, see equations (9) and (10). The time lag for this 'pure fundamentalist' model is then

$$T_r = (c_1 c_2)^{-1} = (T_b \omega_0^2)^{-1} = 0.198 \text{ [trading days]} \approx 1.5 \text{ hours} \tag{15}$$

(Other reasonable parameter values leading to other values of T_r have been tried out, and it turns out that different choices are not critical for the analysis and conclusions.) With $K_b = 0$, all agents have the same perfect information about p_r. Any change in p_r is responded to by each agent in the same manner. However, action is assumed to be dispersed in time. While such agents receive perfect information, they do not act instantaneously, and slower than bandwagon agents. With $K_b = 0$, a small upwards jump in p_r, for instance because of a reduction in the Central Bank overnight rate, results in a price adjustment path that is a first order stable exponential step response as shown in figure 14:

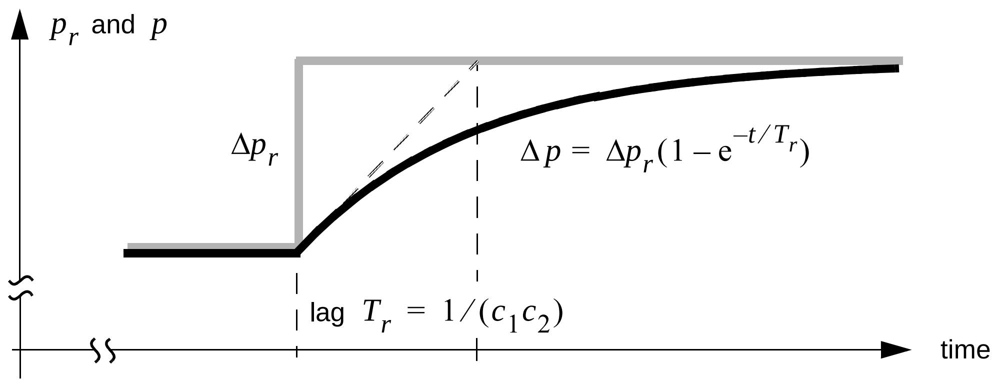


Figure 14

We have no overshoot, no oscillations, no unpredictable excursions, just a smooth and asymptotically perfect price adjustment. This is of course a completely unrealistic representation of what occurs in the real world. At the same time, it should be noted that this is the way a stock market *ought to* work, reflecting real-economic changes impacting the price, and nothing else.

Now to the long-range model, but without crashes, as shown in figure 4. First, choices were made for the recognition time lag in connection with long-term price change;

$T_f = 200$, and for the forgetting time in the optimism subsystem; $1/c_3 = 714$. These choices are fairly arbitrary, but they should be chosen within the range of one to three years (one 'trading year' is defined to be 250 trading days). Several simulations show that values over this range give the same qualitative behaviour. The gain $c_4 = 234046$ was determined by demanding that the period of one long-range cycle should be in the order of 8 years = 2000 trading days. An increased c_4 leads to shorter periods, and vice versa[8]. Finally, the nonlinear 'decoupling function for low prices' shown in figure 6,

$$f(p) = \frac{1}{1 + (p_r/p)^{c_6}} \tag{16}$$

was determined by experimenting, ending with a choice $c_6 = 2$ that gave a smooth exponential rise in the upswing part of the cycle, and reasonable peak and bottom prices. It turned out that system dynamics were bounded and cyclical for a wide range of values, but the cycles were rather big in amplitude and 'squarish' for larger values of c_6 .

8. The period should probably be made longer. This will be done in a future version of this paper. But it will not make any qualitive difference for the analysis, it is just a question of some scaling in time for the long-term part of the model.

References

Andresen, Trond (2000-04-14) (in Norwegian) 'Et farlig aksjemarked – og et reformforslag' ('A dangerous stock market – and a reform proposal'), Aftenposten (The Evening Post), Oslo.

Davidson, Paul (1998) 'Financial Markets and the Speculator', Economic Issues ,vol. 3, September, University of East London, UK.

Johansen, A. and Sornette, D. (2000) 'The Nasdaq crash of April 2000: Yet another example of log-periodicity in a speculative bubble ending in a crash', The European Physical Journal B, no. 17.

Jones , C. M. and Seguin, P. (1997) 'Transaction Costs and Price Volatility: Evidence from Comission Deregulation', American Economic Review, vol. 87 no. 4.

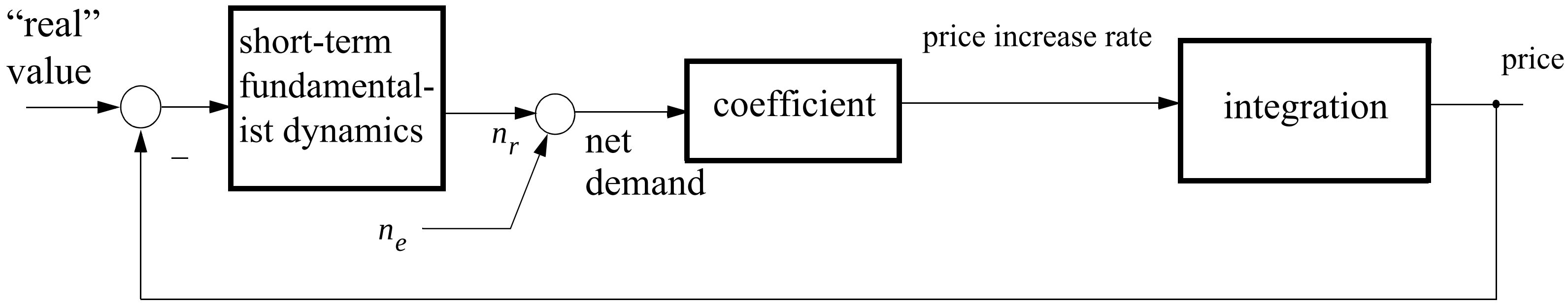
Keynes, John Maynard ([1936]1973) 'The General Theory of Employment, Interest and Money', Macmillan, Cambridge University Press, UK.

Lux, Thomas and Marchesi, Michele (2000) 'Volatility Clustering in Financial Markets: A Micro-Simulation of Interacting Agents', International Journal of Theoretical and Applied Finance, Vol. 3.

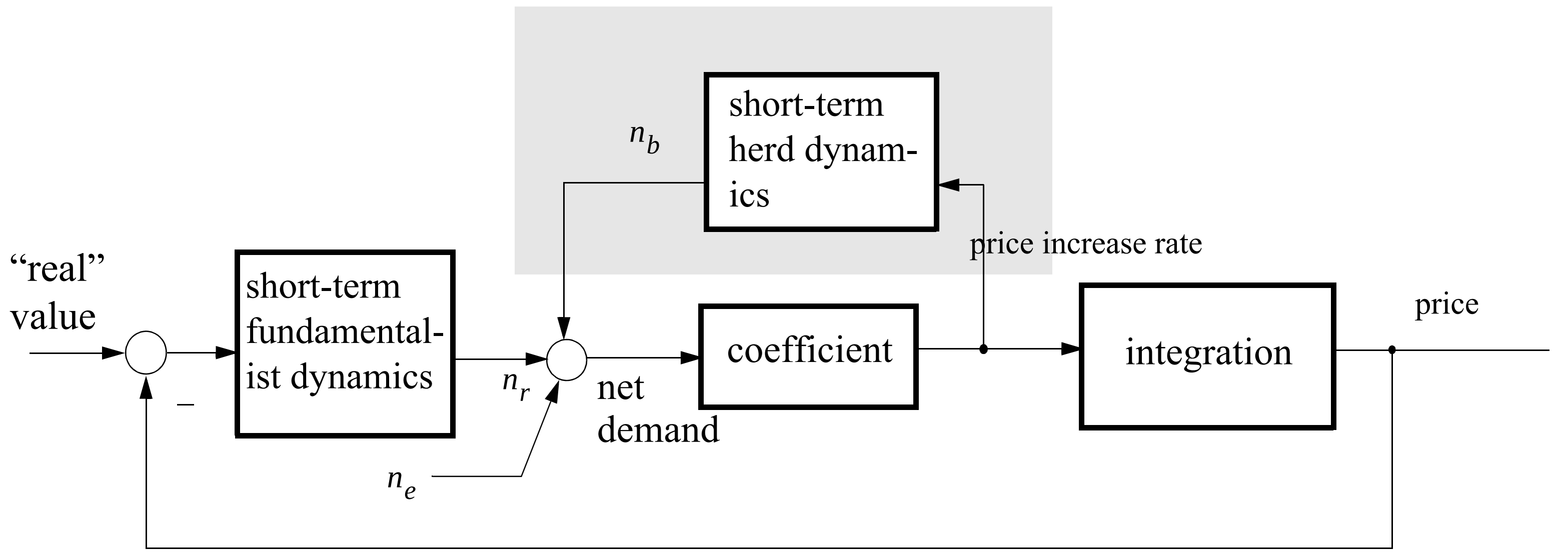
Report (2001-04-20) (in Norwegian): 'Selv superinvestorene skjelver' ('Even the super investors tremble'), Aftenposten (The Evening Post), Oslo.

An ideal market – without herd mentality

(it is hardly volatile at all)

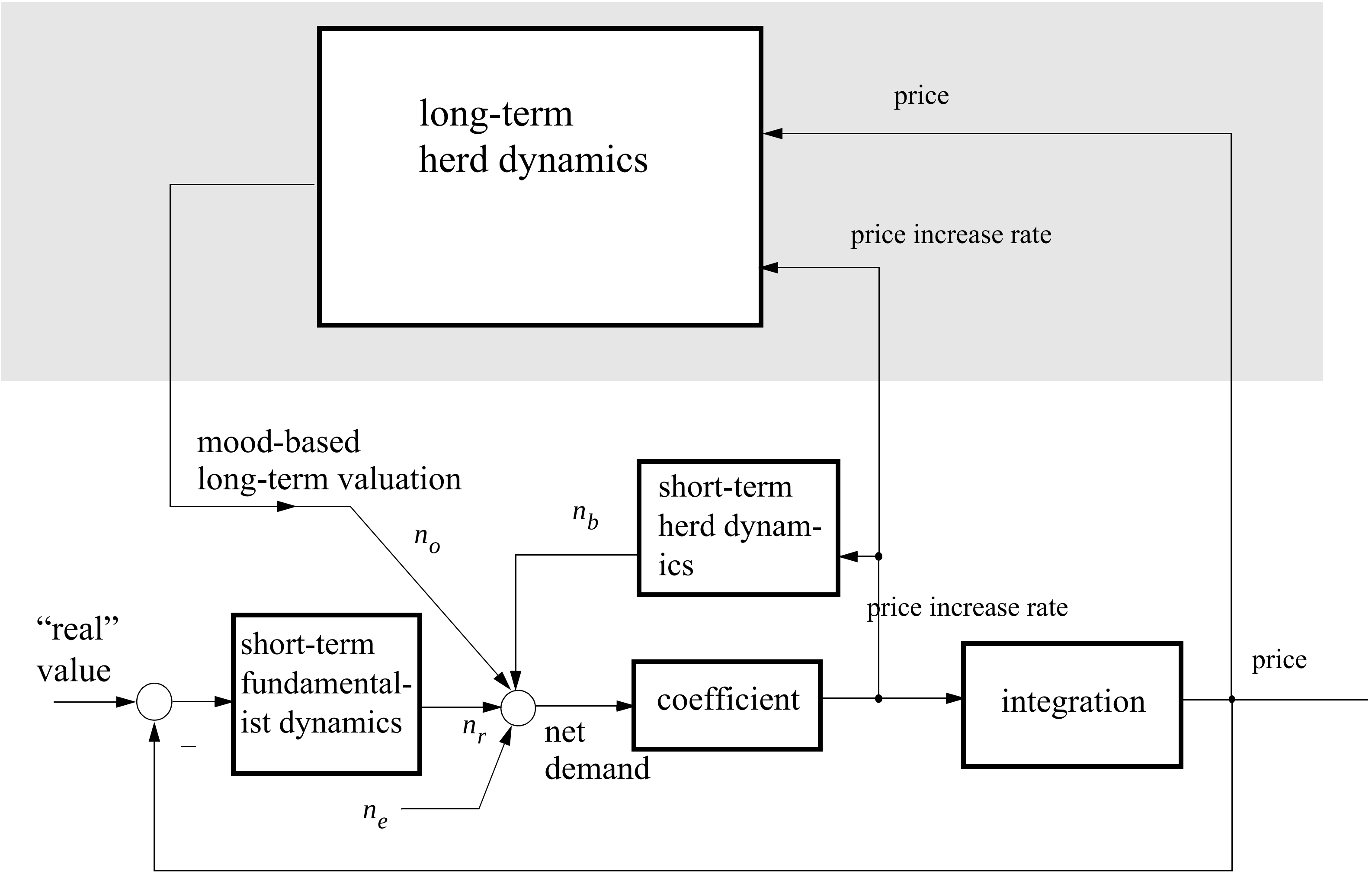


A market with short-term herd mentality
(is *much* more volatile)



A market also with long-term herd mentality

(is volatile and also slowly oscillating with large swings)



Reform idea:
a countercyclical fee