Capitalists can enjoy a persistent profit flow in an economy with no injection of fresh money

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Abstract

There is a large and elaborate literature in economics about the (in)feasibily of capitalists in the aggregate enjoying a stable profit. Many conditions have been put forward for this to be feasible, for instance that extra ("fresh") money must be persistently added to the system, typically in the form of bank credit. This brief note argues that this is not necessary, and that this is very simple to conclude by using a continuous time linear model of a closed economic circuit. The paper also explains Marx' m-c-m' puzzle. Furthermore it argues that a constant – not falling – profit rate is feasible, and that this profit rate is independent of capitalists' share of output.

1 Introduction

In this brief note it will be argued that capitalists' profits and profit rate can be achieved and held stably at some value above zero in an economy, without the need of injection of additional money. The model to be presented is very simple. There is no foreign sector, no government and no banks, only households and non-financial firms in a circuit where the circulating amount of money is constant. All investment is done by capitalists recycling part of their profits. It will be seen that even such a simple model is sufficient to make the main arguments.

Output is shared between workers and capitalists. There are two lags in the model, one for firms and one for households. (Capitalists could have had a lag too, but this is not necessary for our argument.) Workers consume all their wages. Capitalists receive profits, consume a share of this, and invest the rest.

2 The model

Readers are recommended to check out the block diagram below, figure 1. All needed information is contained in that diagram.The model, however, will initially be presented based on equations. We will see that it boils down to a three-state linear dynamical system.

We define the following variables and parameters; denominations are indicated in brackets:

T_F = first order time lag for the aggregate of non-financial firms $[y]$.

T_W = first order time lag for the aggregate of (non-saving) workers/households $[y]$.

$Y_O(t)$ = aggregate income to be shared between workers and capitalists $[\$/y]$.

$Y_D(t)$ = aggregate demand to firms $[\$/y]$.

$K(t)$ = capitalists' accumulated capital [\$].

d = depreciation rate on K $[1/y]$.

r = profit rate $[1/y]$.

π = share of aggregate income that capitalists receive [ ]; $0 < \pi < 1$.
The workers' share is then $1 - \pi$.

s = share of capitalists' profit flow that is invested, not used for consumption [ ]; $0 < s < 1$.

$I(t)$ = flow of investment [\$/y]

$W(t)$ = workers' wages flow [\$/y];
workers are assumed to use their entire wages for consumption = $C_W(t)$, with a lag T_W.

$\Pi(t)$ = profit flow to capitalists [\$/y].

$C_K(t)$ = capitalists' aggregate consumption flow [\$/y].

$C(t)$ = total consumption flow [\$/y], we have $C = C_W + C_K$.

We start the presentation with the input/output dynamics of two defined aggregates: firms and worker households. These dynamics may be explained via the firm aggregate which has time lag T_F – properties for the worker aggregates is similar, except for the time lag being T_W. We assume that all lags are of the first-order type, corresponding to a differential equation (using the aggregate of firms):

$$T_F \dot{Y}_o = -Y_O + Y_D \tag{1}$$

The money held at any time[1] by the aggregate of firms, must satisfy

$$\dot{M}_F = -Y_O + Y_D \ , \tag{2}$$

so that

$$Y_O = \frac{M_F}{T_F} \ (= M_F v_F) \tag{3}$$

where v_F is firm money velocity [$1/y$] (but we will not use v_F in the following). With such input-output dynamics, a stepwise change in the input flow gives an output response that adjusts asymptotically to the input in the form of a stable exponential with a lag T_F. This type of subsystem (= aggregate) is of order 1. The worker household subsystem has the same properties. Equations are:

$$T_W \dot{C}_W = -C_W + W \tag{4}$$

with

$$C_W = \frac{M_W}{T_W} \tag{5}$$

The last differential equation is for capital accumulation and depreciation:

$$\dot{K} = -dK + I \tag{6}$$

which is not part of the circuit but only a measure of success seen from the capitalists' position.

To complete the model, we need some additional (non-differential-) equations. The profit flowing to capitalists is:

$$\Pi = \pi Y_O \tag{7}$$

For workers' wages we have their share of output:

$$W = (1 - \pi)\, Y_O \tag{8}$$

For demand to firms we have:

$$Y_D = C + I \tag{9}$$

where

$$C = C_K + C_W = (1 - s)\Pi + C_W, \tag{10}$$

and

$$I = s\Pi, \tag{11}$$

This completes the set of equations describing the system.

The model until now described through a set of equations, is shown in figure 1 as a block diagram:

[1] Obviously circulating money stock must reside somewhere at any time. And for money velocity not to be infinite, money has to stay with an aggregate for a finite time. This is accounted for by the time lag in the first order differential equation representation.

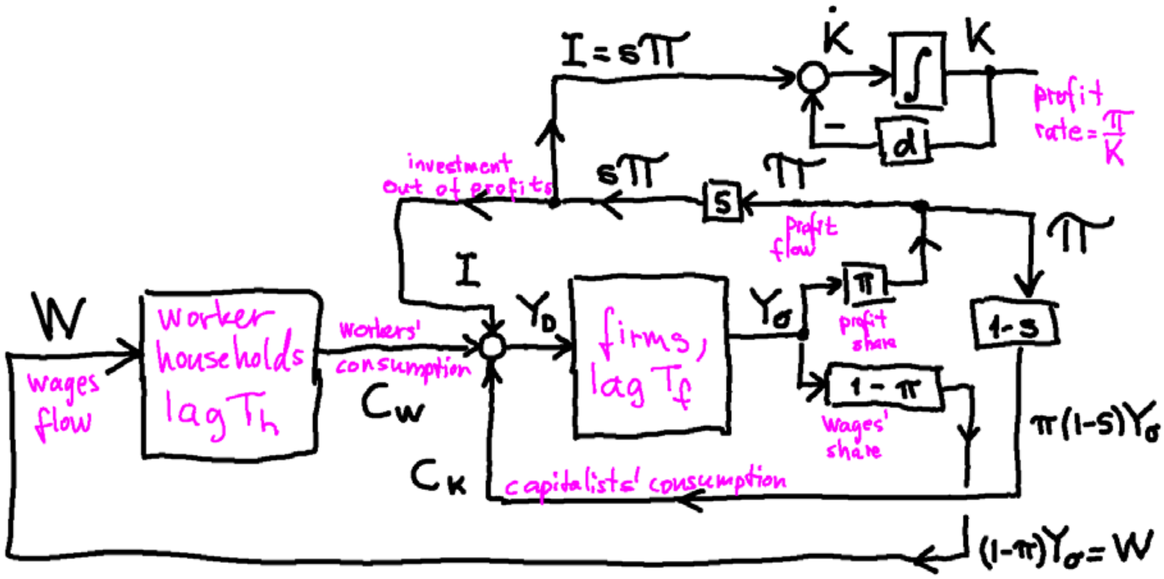

Figure 1: Block diagram for monetary circuit with capitalists and workers.

3 The argument

We now wish to check whether there exists a stationary system state where the aggregate profit flow is positive and constant, and what the condition(s) for this are. The reader is forewarned that the exercise is exceedingly simple, compared to the amount of research and discussion that has been done about this issue. This begs the question of whether I have completely misunderstood or overlooked something. Anyway, here goes:

Since the system is in equilibrium (= stationary), all derivatives are zero. From (2) we get the trivial result

$$Y_O = Y_D = Y \tag{12}$$

Capitalists invest the flow $I = s\Pi = s\pi Y$, cf. (11) and (7). They extract the flow $\Pi = \pi Y$. In Marxian terms, as long as $s < 1$, i.e. capitalists consume some of their profits, $m' > m$! It can't be simpler, and may be compared to the incredibly convoluted discussions among marxists and Marxian economists about this.

Turning now to the profit rate, and setting the left side of (6) to zero, we get

$$dK = I = s\Pi = s\pi Y \Longrightarrow K = \frac{s\pi Y}{d} \tag{13}$$

leading to the equilibrium profit rate

$$r = \frac{\Pi}{K} = \frac{\pi Y d}{s\pi Y} = \frac{d}{s} \tag{14}$$

We have the interesting result that capitalists' profit rate in equilibrium is *not* dependent upon their profit share π of output. And the higher capitalists' consumption out of profits is (i.e. small s), the higher the profit rate. Capitalists also decide the profit rate in the sense that it is higher with a higher depreciation rate.

So, may these questions be resolved *that* simply?
If not, why?